

Solving Rational Inequalities

- (1) Rearrange all terms to one side, the other side zero.
*** do NOT cross multiply ***
- (2) Write expression with single, common denominator, removing any common factors (holes).
- (3) Determine any vertical asymptotes from the simplified denominator.
- (4) Determine any zeroes from the simplified numerator.
- (5) Use an interval table (sign chart) to check for positive and negative intervals.
- (6) Write your solution, keeping in mind ALL restrictions (holes, VAs, real-world considerations).

Oct 19-7:38 PM

Why we CANNOT use cross multiplication on rational inequalities:

For a rational equation: $\frac{x+3}{x+1} = \frac{x-2}{x-3}$

$$(x+1)(x-3)\frac{x+3}{x+1} = \frac{x-2}{x-3}(x+1)(x-3)$$

$$(x-3)(x+3) = (x-2)(x+1)$$

For a rational inequality:

Because there is a variable involved, we don't know if we multiplied by a positive or negative. Should direction of the inequality change?

$$\frac{x+3}{x+1} \geq \frac{x-2}{x-3}$$

$$(x-3)(x+3) \geq (x-2)(x+1)$$

OR?

$$(x-3)(x+3) \leq (x-2)(x+1)$$

Oct 21-9:19 AM

Ex.1 Solve $x - 1 < \frac{12}{x}$ (see p.291 for similar example)

$$x - 1 - \frac{12}{x} < 0$$

$$\frac{x(x-1) - 12}{x} < 0$$

$$\frac{x^2 - x - 12}{x} < 0$$

$$\frac{(x-4)(x+3)}{x} < 0$$

$x \neq 0$
VA: $x = 0$
zeros: $x = -3, 4$

	-3	0	4	
$x-4$	-	-	-	+
$x+3$	-	+	+	+
x	-	-	+	+
result	-	+	-	+

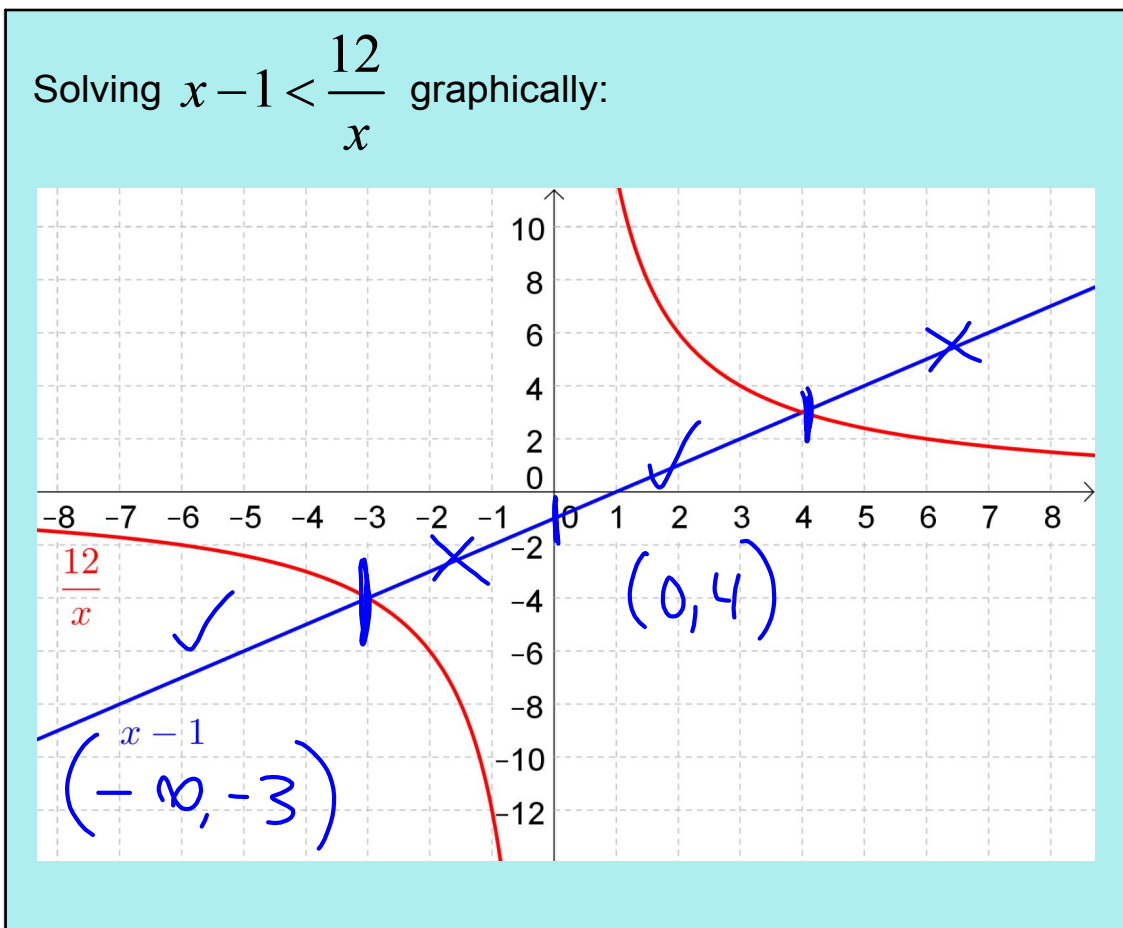
pass *pass*

OR

$$x \in (-\infty, -3) \cup (0, 4)$$

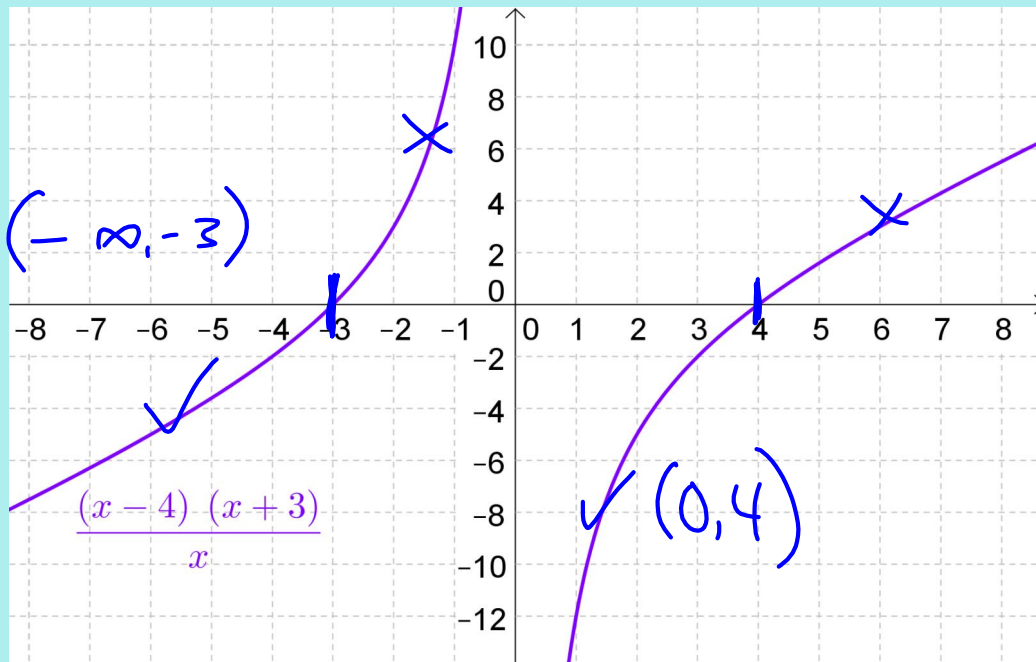
$$x < -3 \text{ OR } 0 < x < 4$$

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Solving $\frac{(x-4)(x+3)}{x} < 0$ graphically:



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Ex.2 Solve $\frac{x}{x-1} < \frac{5x-3}{(x-1)(x+1)}$

$$\frac{x}{x-1} - \frac{5x-3}{(x-1)(x+1)} < 0$$

$$\frac{x(x+1) - (5x-3)}{(x-1)(x+1)} < 0$$

$$\frac{x^2 + x - 5x + 3}{(x-1)(x+1)} < 0$$

$$\frac{x^2 - 4x + 3}{(x-1)(x+1)} < 0$$

$$\frac{(x-3)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} < 0$$

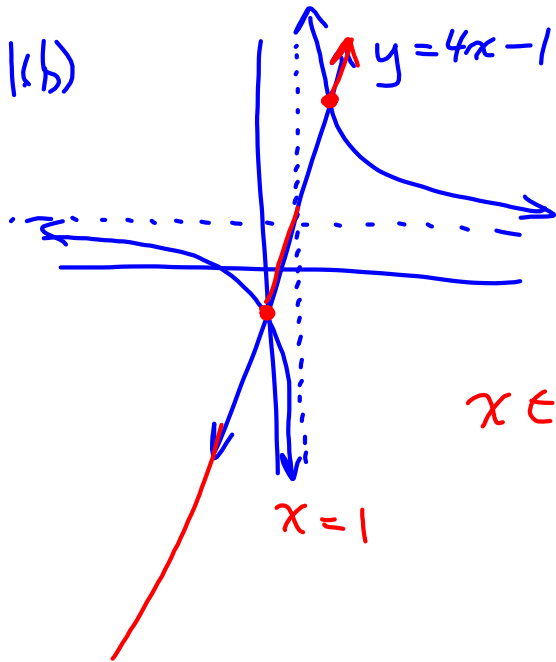
$$\frac{x-3}{x+1} < 0, \quad x \neq 1, x \neq -1$$

$$x \neq 1$$

Oct 21-10:01 AM

Assigned Work:

p.295 # 1, 4acf, 5ace, 7, 9, 13?
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y-value of 0
 $4x-1 > \frac{x+5}{x-1}$

$x \in (-0.5, 1) \cup (2, \infty)$

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4(c) $\frac{3}{x-2} < \frac{4}{x}$

$$\frac{3}{x-2} - \frac{4}{x} < 0$$

$$\frac{3x - 4(x-2)}{x(x-2)} < 0$$

$$\frac{-x + 8}{x(x-2)} < 0$$

$$\frac{-(x-8)}{x(x-2)} < 0$$

VA: $x=0, 2$
 Zeros: $x=8$

	0	2	8	
(-1)	-	-	-	-
(x-8)	-	-	-	+
(x)	-	+	+	+
(x-2)	-	-	+	+
	+	-	+	-

pass pass

$x \in (0, 2) \cup (8, \infty)$

Oct 20-2:01 PM

5(4)

$$\frac{6t^2 - 5t + 1}{2t + 1} > 0$$

$$\frac{(2t - 1)(3t - 1)}{2t + 1} > 0$$

z: $\frac{1}{2}, \frac{1}{3}$
VA: $-\frac{1}{2}$

$2t - 1$		$3t - 1$		$2t + 1$		

-1 - $\frac{1}{2}$ 0 $\frac{1}{3}$ 0.4 $\frac{1}{2}$

$\frac{2}{6}$ $\frac{3}{6}$
 $\frac{4}{12}$ $\frac{5}{12}$ $\frac{6}{12}$

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9. $f(t) = \frac{5t}{t^2 + 3t + 2}$ tap water
 $g(t) = \frac{15t}{t^2 + 9}$ pond water

$f(t) > g(t)$

$$\frac{5t}{(t+1)(t+2)} > \frac{15t}{t^2 + 9}$$

$$\frac{5t}{(t+1)(t+2)} - \frac{15t}{t^2 + 9} > 0$$

$$\frac{5t(t^2 + 9) - 15t(t+1)(t+2)}{(t+1)(t+2)(t^2 + 9)} > 0$$

$$\frac{5t^3 + 45t - 15t^3 - 45t^2 - 30t}{(t+1)(t+2)(t^2 + 9)} > 0$$

$$\frac{-10t^3 - 45t^2 + 15t}{(t+1)(t+2)(t^2 + 9)} > 0$$

$$\frac{-5t(2t^2 + 9t - 3)}{(t+1)(t+2)(t^2 + 9)} > 0$$

$D = b^2 - 4ac$
 $= 81 + 24$
 $= 105$

$$t = \frac{-9 \pm \sqrt{105}}{4}$$

VA: ~~$t < -1$~~ , ~~$t < -2$~~

zeros: $t = 0$, $t = \frac{-9 + \sqrt{105}}{4}$, $t = \frac{-9 - \sqrt{105}}{4}$

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-2		-1		0		$\frac{-9 + \sqrt{105}}{4}$		$\frac{-9 - \sqrt{105}}{4}$		

$t \in$

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$$\left| \frac{x}{x-4} \right| \geq 1$$

$$\textcircled{1} \frac{x}{x-4} \geq 1$$

OR

$$\textcircled{2} \frac{x}{x-4} \leq -1$$

$$|x| \geq 1$$



$$x \geq 1$$

OR

$$x \leq -1$$

Oct 20-2:26 PM