

Transformations of Trigonometric Functions

Oct 28/2016

In general:  $y = af[k(x - p)] + q$ 

For trigonometric functions, this becomes

$$y = a \sin[k(x - p)] + q$$

or

$$y = a \cos[k(x - p)] + q$$

or

$$y = a \tan[k(x - p)] + q$$

$$y = a \frac{\sin[k(x-p)]}{\cos[k(x-p)]} + q$$

May 17-9:17 AM

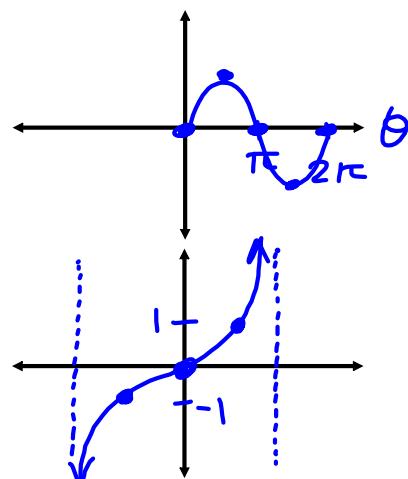
## (1) Graphing From Key Points

For sine and cosine, use points from the x- and y-axes on the unit circle.

$$\theta \in \left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \right\}$$

For tangent, use a cycle between two vertical asymptotes:

$$\theta \in \left\{ -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2} \right\}$$



Transform each point:

and VA

$$(x, y) \rightarrow \left( \frac{x}{k} + p, ay + q \right)$$

Oct 29-8:47 PM

## (2) Graphing From Key Properties

a - vertical reflection and amplitude

k - horizontal reflection and period

$$\text{sine and cosine: } \text{period} = \frac{2\pi}{k}$$

$$\text{tangent: } \text{period} = \frac{\pi}{k}$$

p - phase shift of starting point (from  $x = 0$ )

q - axis of the curve,  $y = q$

May 17-9:18 AM

## (3) Determining Equations of Transformed Functions

1. Select parent function (sine or cosine, tangent).
2. Identify key properties of period and axis of the curve and use them to determine k and q.
3. Identify key properties of amplitude and phase shift and use them to determine a and p.
4. Write the equation, remembering that multiple answers may correctly represent the same graph.

May 17-9:17 AM

Assigned Work:

p.343 # 1bd, 3, 4, 5, 8ade, 9, 11, 12, 14

b  
c

4.(d) period =  $\frac{2\pi}{k}$        $T = \text{period}$

$$k = \frac{2\pi}{\text{per.}}$$

$$k = \frac{2\pi}{\frac{1}{2}}$$

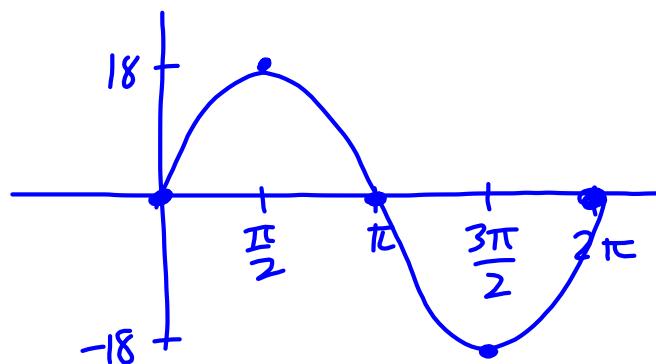
$$= 4\pi$$

$|a| = 11$       Axis:  $y = 0 \rightarrow g = 0$

$$f(x) = 11 \sin(4\pi x)$$

May 22-8:28 AM

5. (a)



Oct 31-12:43 PM

9.  $P(t) = -20 \cos\left(\frac{5\pi}{3}t\right) + 100$

(a)  $k = \frac{5\pi}{3}$        $T = \frac{2\pi}{k}$

$$\begin{aligned} &= \frac{2\pi}{\frac{5\pi}{3}} \\ &= 2\pi^{-1} \left(\frac{3}{5}\right) \\ &= \frac{6}{5} \text{ s} \\ &= 1.2 \text{ s} \end{aligned}$$

$\frac{5\pi}{3}$  (seconds)  
→ radians

period represents time per beat of the person's heart.

(b)  $\frac{60 \text{ s}}{1.2 \text{ s}} = 50 \quad \therefore 50 \text{ beats per minute}$

$R = \{P \in \mathbb{R} \mid 80 \leq P \leq 120\}$

Oct 31-12:44 PM

11.

Graph of  $y = \cos x$  showing two full cycles. The amplitude is 25, the vertical shift is 27, and the period is 3.

$y = \cos x$

$|a| = 25$   
 $a = -25$   
 $g = 27$

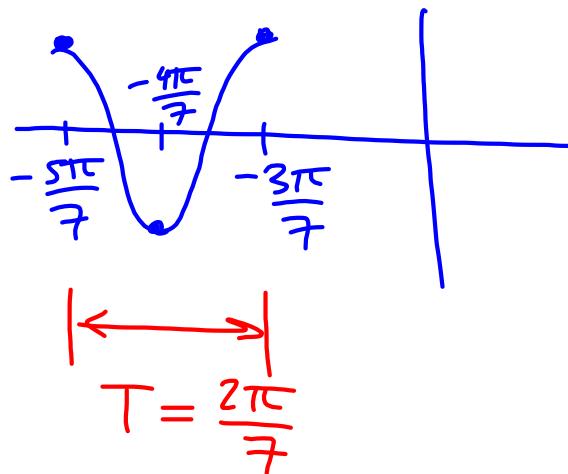
$T = 3$   
 $T = \frac{2\pi}{K}$   
 $K = \frac{2\pi}{T}$   
 $K = \frac{2\pi}{3}$

$P = 0$

$y = -25 \cos\left(\frac{2\pi}{3}t\right) + 27$

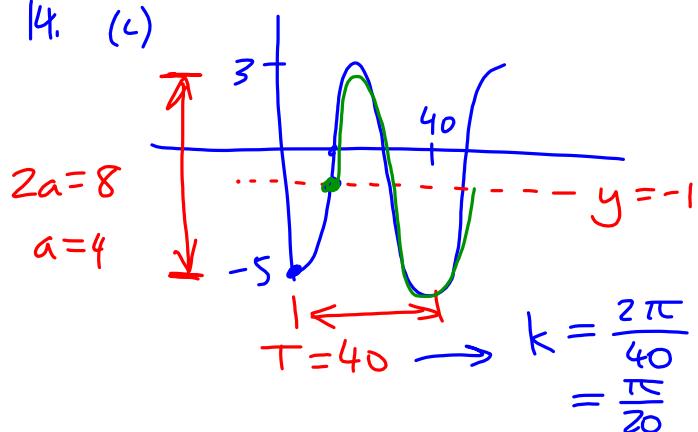
Oct 31-12:57 PM

12.



Oct 31 1:01 PM

14. (c)



$$\text{Axis: } g = \frac{\max + \min}{2} \quad |a| = \frac{\max - \min}{2}$$

or  $y = -4 \cos\left(\frac{\pi}{20}x\right) - 1$

$$y = 4 \sin\left(\frac{\pi}{20}(x-10)\right) - 1$$

Oct 31 1:05 PM

