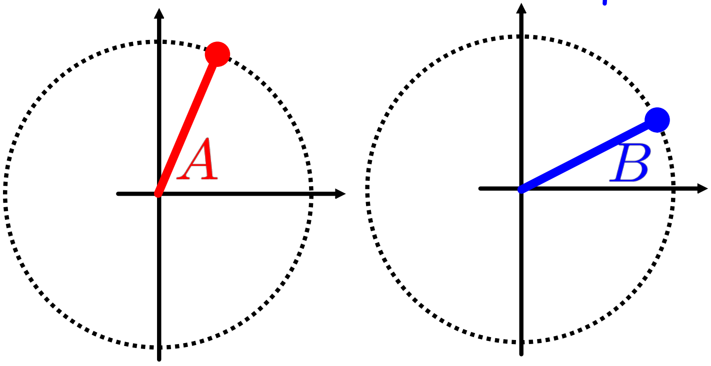
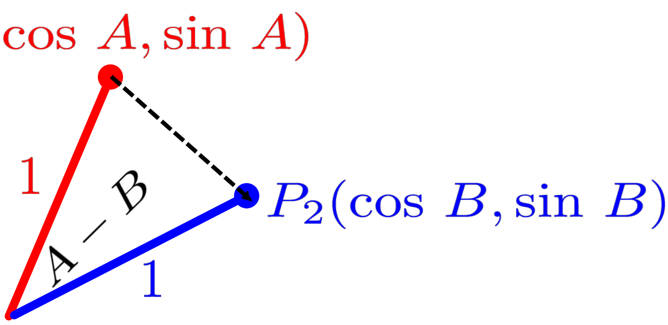
Compound Angle Formulas

Nov. 16/2016

Consider the angles 'A' and 'B' on the unit circle.





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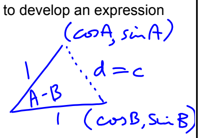
Cosine Law: $c^2 = a^2 + b^2 - 2ab\cos C$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Use cosine law and distance formula to develop an expression in terms of 'A' and 'B':



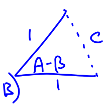
$$d = \sqrt{(\cos B - \cos A)^2 + (\sin B - \sin A)^2}$$

$$= \sqrt{\cos^2 B - 2\cos A\cos B + \cos^2 A + \sin^2 B - 2\sin A\sin B + \sin^2 A}$$

$$= \sqrt{1 + 1 - 2\sin A\sin B - 2\cos A\cos B}$$

$$d = \sqrt{2 - 2\sin A\sin B - 2\cos A\cos B}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



$$c^2 = 1^2 + 1^2 - 2(1)(1)\cos(A-B)$$

$$c^2 = 2 - 2\cos(A-B)$$

$$2\cos(A-B) = 2 - c^2$$, but $c = d$ so $c^2 = d^2$

$$2\cos(A-B) = 2 - 1(2 - 2\sin A\sin B - 2\cos A\cos B)$$

$$2\cos(A-B) = 2\sin A\sin B + 2\cos A\cos B$$

$$\boxed{\cos(A-B) = \cos A\cos B + \sin A\sin B}$$

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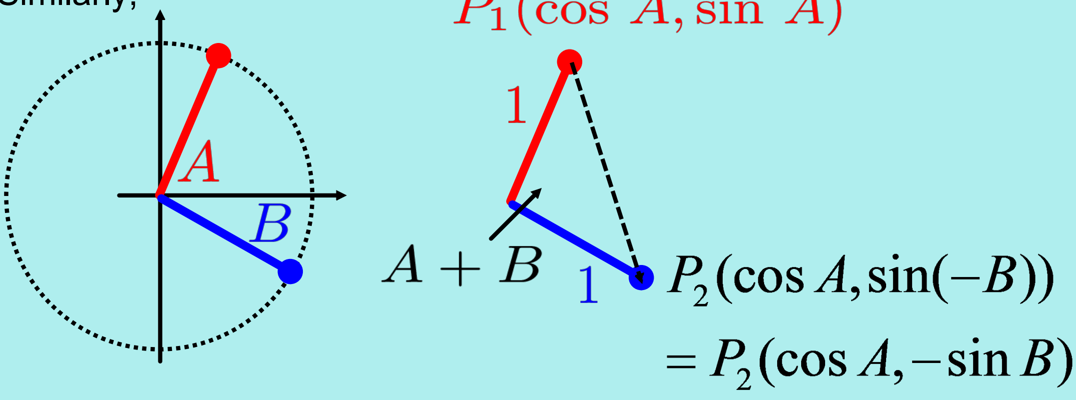
Similarly,



$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

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Similar compound angle formulas can be obtained for sine using the complimentary angle formula:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

Ex.1 $\sin(A+B)$

$= \cos\left(\underbrace{\frac{\pi}{2}} - \underbrace{(A+B)}\right)$

$= \cos\left(\underbrace{\frac{\pi}{2} - A}_{x} - \underbrace{B}_{y}\right)$

$\cos(x-y)$
$= \cos x \cos y + \sin x \sin y$

$= \cos\left(\frac{\pi}{2} - A\right)\cos B + \sin\left(\frac{\pi}{2} - A\right)\sin B$

$= \sin A \cos B + \cos A \sin B$

$$\boxed{\sin(A+B) = \sin A \cos B + \cos A \sin B}$$

$$\boxed{\sin(A-B) = \sin A \cos B - \cos A \sin B}$$

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For the tangent function, use the quotient identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Ex.2 $\tan(A+B)$

$$= \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B} \times \frac{\frac{1}{\cos A\cos B}}{\frac{1}{\cos A\cos B}}$$

$$= \frac{\frac{\sin A\cancel{\cos B}}{\cos A\cancel{\cos B}} + \frac{\cancel{\cos A}\sin B}{\cancel{\cos A}\cos B}}{\frac{\cancel{\cos A\cos B}}{\cancel{\cos A\cos B}} - \frac{\sin A\sin B}{\cos A\cos B}}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}}$$

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Many applications of the compound angle formulas involve angles from the special triangles.

Ex.1 Simplify and then evaluate:

$$\cos\frac{7\pi}{12}\cos\frac{5\pi}{12} + \sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$$

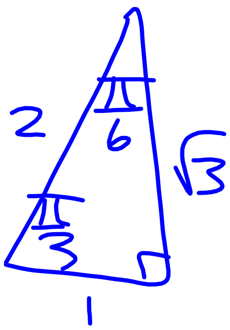
$\cos(A-B)$

$$= \cos\left(\frac{7\pi}{12} - \frac{5\pi}{12}\right)$$

$$= \cos\left(\frac{2\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)$$

$$= \frac{\sqrt{3}}{2}$$



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Ex.2 Determine an exact value for $\tan\left(\frac{-5\pi}{12}\right)$

notes:

(1) simplest to convert to RAA and apply CAST

(2) easier to see sum or difference of special angles by converting to degrees, then back to radians

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Assigned Work:

p.400 # 1-4, 5acf, 6cde, 8, 9ade, 10, 13

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