Proving Trigonometric Identities

Nov 22/2016

An identity is an equation which is always true for all values of the variable(s) within the domain.

To prove a trigonometric identity, manipulate one side of the equality until a form identical to the other side is reached. Any restrictions where information is lost (e.g., dividing out factors) must be noted.

It is also possible to disprove an equality through a counterexample. If the equality can be shown false with a single example, it is considered to be false in general.

Nov 15-5:40 PM

Tips for working with trig identities:

1. Keep your goal in mind! As you work one side, keep in mind how to get closer to your target.

2. Start with the most complicated side and try to make it simpler.

3. If stuck, try expressing in terms of sine and cosine.

4. Only work on one side at a time. Only switch sides if you cannot progress any further (e.g., you are stuck, or you will make it more complicated)

Nov 15-5:48 PM

Ex.1 Prove $\dfrac{\cos(x-y)}{\cos(x+y)} = \dfrac{1+\tan x\ \tan y}{1-\tan x\ \tan y}$

$$LS = \frac{(\cos x \cos y + \sin x \sin y)}{\cos x \cos y - \sin x \sin y} \frac{\times \frac{1}{\cos x \cos y}}{\times \frac{1}{\cos x \cos y}}$$

$$= \frac{1 + \tan x \tan y}{1 - \tan x \tan y}$$

Nov 15-5:56 PM

Some educators prefer a one-sided proof, where only one side of the equality is turned into the other.

This view has merit, as it maintains an important property of a proof like this, called <u>reversibility</u>.

Ex.1 Prove $\dfrac{\cos(x-y)}{\cos(x+y)} = \dfrac{1+\tan x\ \tan y}{1-\tan x\ \tan y}$

Nov 15-5:56 PM

Assigned Work:

p.416 # 5, 8, (9, 10, 11)(odd), 17

a b e h

9(a) $LS = \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta}$

$= \dfrac{(\cos\theta - \sin\theta)\cancel{(\cos\theta + \sin\theta)}}{\cos\theta\cancel{(\cos\theta + \sin\theta)}}$

$= \dfrac{\cos\theta - \sin\theta}{\cos\theta}$

$= \dfrac{\cos\theta}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= 1 - \tan\theta$

Nov 6-9:56 PM

10(b)

$LS = \sin^2\theta + \cos^4\theta$

$= (1-\cos^2\theta) + (\cos^2\theta)^2$

$= 1-\cos^2\theta + (1-\sin^2\theta)^2$

$= 1-\cos^2\theta + (1-2\sin^2\theta + \sin^4\theta)$

$= 2-\cos^2\theta - 2\sin^2\theta + \sin^4\theta$

$= 2-\cos^2\theta - 2(1-\cos^2\theta) + \sin^4\theta$

$= 2-\cos^2\theta - 2 + 2\cos^2\theta + \sin^4\theta$

$= \cos^2\theta + \sin^4\theta$

$(1-\sin^2\theta)(1-\sin^2\theta)$

$=$

Nov 24-2:01 PM

11(e)

$$RS = 2\cot 2\theta$$

$$= \frac{2}{\tan 2\theta}$$

$$= \frac{2}{\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}$$

$$= \frac{\cancel{2}}{1} \times \frac{1-\tan^2\theta}{\cancel{2}\tan\theta}$$

$$= \frac{1-\tan^2\theta}{\tan\theta}$$

$$= \frac{1}{\tan\theta} - \frac{\tan^{\cancel{2}}\theta}{\cancel{\tan\theta}}$$

$$= \cot\theta - \tan\theta$$

Nov 24-2:08 PM

11(h)

$$LS = \csc 2x + \cot 2x$$

$$= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$$

$$= \frac{1+\cos 2x}{\sin 2x}$$

$$= \frac{1+(2\cos^2 x - 1)}{2\sin x\cos x}$$

$$= \frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\,\cancel{\cos x}}$$

$$= \cot x$$

Nov 24-2:12 PM