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The Logarithmic Function

Many phenomena in the natural sciences (physics, chemistry, biology, astronomy) can be described using exponential functions. To solve problems involving a function, it is often useful to use the inverse function.

Ex.1 Given $f(x) = 10^x$

(a) $f(3) = 10^3$

$= 1000$

(b) Solve $f(x) = 100$

$10^x = 100$

$10^x = 10^2$

$\therefore x = 2$

(c) Solve $f(x) = 37$

$10^x = 37$

$10^1 = 10 \quad 10^2 = 100$

try $10^{1.5} \doteq 31.6$

$10^{1.6} \doteq 39.8$

$10^{1.58} \doteq 38.0$

$10^{1.56} \doteq 36.3$

$10^{1.57} \doteq 37.15$

$10^x = 37$

$\log(10^x) = \log 37$

$x = \log_{10} 37$

$x \doteq 1.5682$

$x \rightarrow f(x) \rightarrow y$

$3 \rightarrow f(x) \rightarrow 75$

$75 \rightarrow f^{-1}(x) \rightarrow 3$

Invented by John Napier in the 17th century, logarithmic functions (and the associated tables of values generated using them) were the only effective numerical tools for dealing with exponential functions until the development of computers and calculators.

Some applications of logarithmic functions include:

pH levels (acid/base) in chemistry

exponential growth/decay in biology

sound intensity in physics/music

light intensity & absorption in physics/astronomy

richter scale (earthquakes) in physics/geology

Recall:

(1) To find an inverse, swap x and y.

(2) A function and its inverse <u>undo</u> each other.

Exponential Relation: $y = a^x, a > 0, a \neq 1$

Inverse Relation: $x = a^y$

There is no way to rearrange this algebraically, so we introduce a new representation:

Logarithmic Relation: $y = log_a x, a > 0, a \neq 1$

"log to the base a of x"

$\boxed{y = \log_a x \quad \text{is equivalent to} \quad x = a^y}$

The two most important logarithmic functions have bases of '10' and 'e', so they get special notation:

(1) $\log_{10} x = \log x$ is the "common log".

(2) $\log_e x = \ln x$ is the "natural log".

"lawn of x"

$e \doteq 2.718$ is the "natural number".

$2.718^{4.6052} =$

Ex.2 Write an equivalent exponential expression.

(a) $\log_2 32 = 5$

$a=2$

$x=5$

$y=32$

$32=2^5$

(b) $\log_{10} 1 = 0$

$a=10$

$x=0$

$y=1$

$1=10^0$

$y=a^x$

$x=\log_a y$

Ex.3 Write an equivalent logarithmic expression.

(a) $3^4 = 81$

$\log_3 81 = 4$

(b) $\dfrac{1}{16} = 4^{-2}$

$\log_4\left(\frac{1}{16}\right) = -2$

Assigned Work:

p.451 # 4 to 9