Laws of Logarithms

Recall: Exponent Laws

same base

$$(a^x)(a^y) = a^{x+y}$$

$$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}, \quad a \neq 0$$

$$a^{-x} = \frac{1}{a^x}, \quad a \neq 0$$

$$\left(a^x\right)^y = a^{xy}$$

$$a^0 = 1, \quad a \neq 0$$

different bases

$$\left(ab\right)^x = \left(a^x\right)\left(b^x\right)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, \quad b \neq 0$$

$$\cancel{(a+b)^2 = a^2 + b^2}$$

Dec 2-8:13 PM

Ex. Derive the Product Law for Logarithms $(a^x)(a^y) = a^{x+y}$
(use the product law for exponents)

let $m = a^x$ $\quad n = a^y$

$\log_a m = x$ $\quad \log_a n = y$

$$(a^x)(a^y) = a^{x+y}$$

$$(mn) = a^{(x+y)}$$

$x = a^y$

$y = \log_a x$

$$x + y = \log_a (mn)$$

$$\log_a m + \log_a n = \log_a (mn)$$

$$\boxed{\log_a (mn) = \log_a m + \log_a n}$$

Dec 2-8:24 PM

Ex. Derive the Quotient Law for Logarithms $\frac{a^x}{a^y} = a^{x-y}$
(use the quotient law for exponents)

let $m = a^x$ $n = a^y$

$x = \log_a m$ $y = \log_a n$

$\frac{a^x}{a^y} = a^{x-y}$

$\left(\frac{m}{n}\right) = a^{(x-y)}$ $(x) = a^{(y)}$

$x - y = \log_a\left(\frac{m}{n}\right)$

$\boxed{\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n}$

$\frac{a^x}{a^y} = a^{x-y}$

$\frac{m}{n} = a^{x-y}$ $\log_3 3^2 = 2$

$\log_a\left(\frac{m}{n}\right) = \log_a\left(a^{x-y}\right)$

$\log_a\left(\frac{m}{n}\right) = x - y$

$\boxed{\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n}$

Dec 2-8:24 PM

Ex. Derive the Power Law for Logarithms $\left(a^x\right)^y = a^{xy}$
(use the power law for exponents)

let $m = a^x$

$x = \log_a m$

$\left(a^x\right)^y = a^{xy}$

$m^y = a^{xy}$

$\log_a\left(m^y\right) = \log_a\left(a^{xy}\right)$

$\log_a\left(m^y\right) = xy$

$\boxed{\log_a\left(m^y\right) = y \cdot \log_a m}$

Dec 2-8:24 PM

Laws of Logarithms

product law: $\log_a xy = \log_a x + \log_a y$

quotient law: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

power law: $\log_a x^r = r\log_a x$

Ex.1 Simplify then evaluate (no log calculations!):

(a) $\log_3 6 + \log_3 4.5$

(b) $\log_2 48 - \log_2 3$

(c) $\log_5 \sqrt[3]{25}$

Dec 2-8:18 PM

Ex.1 Simplify then evaluate (no log calculations!):

(a) $\log_3 6 + \log_3 4.5$

$= \log_3(6 \times 4.5) = 3$

$= \log_3 3^3$

(b) $\log_2 48 - \log_2 3$

$= \log_2\left(\frac{48}{3}\right) = 4$

$= \log_2 16$

(c) $\log_5 \sqrt[3]{25}$

$= \log_5(25^{\frac{1}{3}})$

$= \log_5((5^2)^{\frac{1}{3}})$

$= \log_5(5^{\frac{2}{3}})$

$= \frac{2}{3}$

$= \frac{1}{3}\log_5 25$

$= \frac{1}{3} \times 2$

$= \frac{2}{3}$

$\log_a xy = \log_a x + \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^r = r\log_a x$

Dec 2-8:40 PM

Ex.2 Use the power law to show $\log_a x = \dfrac{\log_{10} x}{\log_{10} a}$

$$\boxed{\log_a x^r = r\log_a x}$$

Dec 2-8:34 PM

Ex.3 Rewrite as a single log to a common base:

$$\log 12 + \frac{1}{2}\log 7 - \log 2$$

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^r = r\log_a x$$

$= \log 12 + \log(7^{\frac{1}{2}}) - \log 2$

$= \log(12 \cdot \sqrt{7}) - \log 2$

$= \log\left(\frac{12\sqrt{7}}{2}\right)$

$= \log(6\sqrt{7})$

Dec 2-8:42 PM

Assigned Work:

p.475 # 5 (look past obvious answer!),
6, 7, 9ace, 10ace, 11bde, 17

Dec 2-8:41 PM