Example 1 Express in exponential form.

(a) $\log_2\left(\frac{1}{2}\right) = -1$ (b) $\log_{10} 100\,000 = 5$ (c) $\log_3 z = t$

Solution (a) $\log_2\left(\frac{1}{2}\right) = -1$ (b) $\log_{10} 100\,000 = 5$ (c) $\log_3 z = t$

$2^{-1} = \frac{1}{2}$ $10^5 = 100\,000$ $3^t = z$

Example 2 Express in logarithmic form.

(a) $1000 = 10^3$ (b) $2^{-3} = \frac{1}{8}$ (c) $s = 5^r$

Solution (a) $10^3 = 1000$ (b) $2^{-3} = \frac{1}{8}$ (c) $5^r = s$

$\log_{10} 1000 = 3$ $\log_2\left(\frac{1}{8}\right) = -3$ $\log_5 s = r$

Example 3 Evaluate. (a) $\log_3 81$ (b) $\log_{16} 4$ (c) $\log_{10} 0.0001$

Solution (a) $\log_3 81 = 4$ because $3^4 = 81$

(b) $\log_{16} 4 = \frac{1}{2}$ because $16^{\frac{1}{2}} = 4$

(c) $\log_{10} 0.0001 = -4$ because $10^{-4} = 0.0001$

Example 4 Solve for x.

(a) $\log_2(25 - x) = 3$ (b) $3^{x+2} = 7$

Solution (a) $\log_2(25 - x) = 3$

$2^3 = 25 - x$

$8 = 25 - x$

$x = 17$

(b) $3^{x+2} = 7$

$\log_3 7 = x + 2$

$x = \log_3 7 - 2$

EXERCISE 2

1. Express each equation in exponential form.
   (a) $\log_2 64 = 6$ (b) $\log_5 1 = 0$
   (c) $\log_{10} 0.01 = -2$ (d) $\log_8 4 = \frac{2}{3}$
   (e) $\log_8 512 = 3$ (f) $\log_2\left(\frac{1}{16}\right) = -4$
   (g) $\log_a b = c$ (h) $\log_r v = w$
2. Express each equation in logarithmic form.
   (a) $2^3 = 8$ (b) $10^5 = 100\,000$
   (c) $10^{-4} = 0.0001$ (d) $81^{\frac{1}{2}} = 9$
   (e) $4^{-\frac{3}{2}} = 0.125$ (f) $6^{-1} = \frac{1}{6}$
   (g) $r^s = t$ (h) $10^m = n$

3. Evaluate.

(a) $\log_6 6^4$ (b) $\log_2 32$
(c) $\log_4 64$ (d) $\log_8 8^{17}$
(e) $\log_9 9$ (f) $\log_6 1$
(g) $\log_3\left(\frac{1}{27}\right)$ (h) $\log_4 8$
(i) $\log_8 0.25$ (j) $\log_9 \sqrt{3}$

4. Solve each equation for x.

(a) $\log_2 x = 10$ (b) $\log_5 x = 4$
(c) $\log_{10}(3x + 5) = 2$ (d) $\log_3(2 - x) = 3$
(e) $2^{1-x} = 3$ (f) $3^{2x-1} = 5$
(g) $\log_2(\log_3 x) = 4$ (h) $10^{5^x} = 3$

Laws of Logarithms

> Suppose that $x > 0$, $y > 0$, and r is any rational number. Then
> 1. $\log_b(xy) = \log_b x + \log_b y$
> 2. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
> 3. $\log_b(x^r) = r \log_b x$

Example 1 Use the Laws of Logarithms to rewrite the following.

(a) $\log_2(6x)$ (b) $\log_5 x^3y^6$ (c) $\log_{10} \frac{ab}{\sqrt[3]{c}}$

Solution (a) $\log_2(6x) = \log_2 6 + \log_2 x$

(b) $\log_5(x^3y^6) = \log_5 x^3 + \log_5 y^6 = 3 \log_5 x + 6 \log_5 y$

(c)
$$\log_{10} \frac{ab}{\sqrt[3]{c}} = \log_{10} ab - \log_{10} \sqrt[3]{c}$$
$$= \log_{10} a + \log_{10} b - \log_{10} c^{\frac{1}{3}}$$
$$= \log_{10} a + \log_{10} b - \tfrac{1}{3} \log_{10} c$$

Example 2 Express $3 \log_2 s + \frac{1}{2} \log_2 t - 4 \log_2(t^2 + 1)$ as a single logarithm.

Solution
$$3 \log_2 s + \tfrac{1}{2} \log_2 t - 4 \log_2(t^2 + 1) = \log_2 s^3 + \log_2 t^{\frac{1}{2}} - \log_2(t^2 + 1)^4$$
$$= \log_2(s^3t^{\frac{1}{2}}) - \log_2(t^2 + 1)^4$$
$$= \log_2\left(\frac{s^3\sqrt{t}}{(t^2 + 1)^4}\right)$$

EXERCISE 3

1. Use the Laws of Logarithms to rewrite each expression in a form with no logarithms of products, quotients, or powers.

(a) $\log_2 x(x - 1)$
(b) $\log_5\left(\frac{x}{2}\right)$
(c) $\log_2(AB^2)$
(d) $\log_6 \sqrt[4]{17}$
(e) $\log_3(x\sqrt{y})$
(f) $\log_2(xy)^{10}$
(g) $\log_5 \sqrt[3]{x^2 + 1}$
(h) $\log_b \frac{x^2}{yz^3}$
(i) $\log_{10} \frac{x^3y^4}{z^6}$
(j) $\log_{10} \frac{a^2}{b^4 \sqrt{c}}$

2. Evaluate.

(a) $\log_5 \sqrt{125}$
(b) $\log_2 112 - \log_2 7$
(c) $\log_{10} 2 + \log_{10} 5$
(d) $\log_{10} \sqrt{0.1}$
(e) $\log_4 192 - \log_4 3$
(f) $\log_{12} 9 + \log_{12} 16$

3. Rewrite each expression as a single logarithm.

(a) $\log_{10} 12 + \frac{1}{2} \log_{10} 7 - \log_{10} 2$
(b) $\log_2 A + \log_2 B - 2 \log_2 C$
(c) $\log_5(x^2 - 1) - \log_5(x - 1)$
(d) $4 \log_2 x - \frac{1}{3} \log_2(x^2 + 1) + \log_2(x - 1)$
(e) $\frac{1}{2}[\log_5 x + 2 \log_5 y - 3 \log_5 z]$
(f) $\log_a b + c \log_a d - r \log_a s$

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EXERCISE 8.3

B **1.** Most scientific calculators have keys for both LN and LOG ($= \log_{10}$). Use such a calculator to draw the graphs of $y = \ln x$ and $y = \log_{10} x$, $0.1 \leqslant x \leqslant 10$, on the same axes.

2. Graph each function, not by plotting points, but by starting from the graphs of $y = \log_2 x$, $\log_{10} x$, and $\ln x$ given in this section and using transformations. State the domain, range, and asymptote of each function.

(a) $f(x) = \log_2(x - 4)$
(b) $f(x) = -\log_{10} x$
(c) $g(x) = \log(-x)$
(d) $g(x) = \ln(x + 2)$
(e) $y = 2 + \log_{10} x$
(f) $y = \log_2(x - 1) - 2$
(g) $y = 1 - \ln x$
(h) $y = 1 + \ln(-x)$
(i) $y = |\ln x|$
(j) $y = \ln|x|$

3. Evaluate without using a calculator.

(a) $e^{\ln 5}$
(b) $\ln e^2$
(c) $2 \ln e$
(d) $e^{5 \ln 2}$
(e) $\ln \sqrt{e}$
(f) $\ln 2 + 2 \ln 3 - \ln 18$

4. Solve for x.

(a) $e^x = 4$
(b) $\ln x = 6$
(c) $\ln(2x - 1) = 1$
(d) $e^{3x+5} = 10$
(e) $\ln(e^{3-x}) = 8$
(f) $\ln x = \ln 4 + \ln 7$
(g) $\ln(\ln x) = 2$
(h) $e^{e^x} = 5$

5. Find the solution of each equation correct to six decimal places.

(a) $\ln(x + 1) = 3$
(b) $e^{-x} = \frac{1}{2}$
(c) $e^{5x+3} = 10$
(d) $2^{x-5} = 3$

6. Express as a single logarithm.

(a) $\frac{1}{3} \ln x + 2 \ln(3x - 5)$

(b) $2 \ln x - \frac{1}{2} \ln(x^2 - 1) + 3 \ln(x^2 + 1)$

7. Find the domain of each function.

(a) $f(x) = \log_{10}(2 + 5x)$
(b) $f(x) = \log_2(10 - 3x)$
(c) $g(x) = \log_3(x^2 - 1)$
(d) $g(x) = \ln(x - x^2)$
(e) $h(x) = \ln x + \ln(2 - x)$
(f) $h(x) = \sqrt{x - 2} - \ln(10 - x)$

8. Compare the domains of the functions $f(x) = \ln x^2$ and $g(x) = 2 \ln x$.

9. Find each limit.

(a) $\lim_{x \to -4^+} \ln(x + 4)$
(b) $\lim_{x \to \infty} \ln(x + 4)$
(c) $\lim_{x \to 1^+} \log_{10}(x^2 - x)$
(d) $\lim_{t \to \pi^-} \ln(\sin t)$

3. A cell of the bacterium Escherichia coli in a nutrient broth medium divides into two cells every 20 min. Suppose that there are initially 500 cells. Find
(a) the number of cells after t hours
(b) the number of cells after 8 h
(c) the time required for the size to reach 6000 cells

4. The count in a bacteria culture was 5000 after 15 min and 40 000 after 1 h.
(a) What was the initial size of the culture?
(b) Find the population after t hours.
(c) Find the rate of growth after 15 min.
(d) When will the size of the population be 150 000?

5. The population of a certain city grows at a rate of 4% per year. The population in 1980 was 275 000.
(a) What was the population in 1985?
(b) Predict the population in the year 2000, assuming the growth rate remains constant.

6. The population of the world is doubling about every 35 a. In 1987 the total population reached 5 billion.
(a) Find the projected world population
(i) for the year 2001 (ii) for the year 2100
(b) When will the world population reach 50 billion?

7. Uranium-238 has a half-life of 4.5×10^9 a.
(a) Find the mass that remains from a 100 mg sample after t years.
(b) Find the mass that remains from this sample after 10 000 a.
(c) Find the rate of decrease of the mass after 10 000 a.

8. An isotope of sodium, ^{24}Na, has a half-life of 15 h. A sample of this isotope has a mass of 2 g.
(a) Find the mass that remains after t hours.
(b) Find the mass that remains after 5 h.
(c) Find the rate of decrease of the mass after 5 h.
(d) How long will the sample take to decay to a mass of 0.4 g?

9. Uranium-234 has a half-life of 2.5×10^5 a.
(a) Find the amount remaining from a 10 mg sample after a thousand years.
(b) How long would it take this sample to decompose until its mass is 7 mg?

10. A sample of Bismuth-210 decayed to 33% of its original mass after eight days.
(a) Find the half-life of this element.
(b) Find the mass remaining after twelve days.

Key Concepts

- For $a \in (1, \infty)$, $p, q \in (0, \infty)$, $c \in (-\infty, \infty)$,
  a) Product Law: $\log_a (pq) = \log_a p + \log_a q$
  b) Quotient Law: $\log_a \frac{p}{q} = \log_a p - \log_a q$
  c) Power Law: $\log_a (p^c) = c \log_a p$.
- $\log x$ means $\log_{10} x$.
- To change the base of a logarithm from b to a, use the change of base formula $\log_b x = \frac{\log_a x}{\log_a b}$.

Communicate Your Understanding

1. Is it possible to use the quotient law of logarithms to evaluate $\log \frac{-7}{4}$? Explain.
2. Is it possible to use the product law of logarithms to evaluate $\log_2 7 + \log_3 8$? Explain.
3. Explain why the base must be changed to evaluate $\log_5 11$ using a calculator.
4. Is there more than one way to evaluate $\log_3 9 + \log_3 3$? Explain.
5. Is $\log_3 5$ equal to $\log_5 3$? Explain.

Practise

A

1. Copy and complete the table.

Single Logarithm	Sum or Difference of Logarithms
$\log_2 (12 \times 5)$	
	$\log_4 2 + \log_4 11$
$\log_6 (kg)$	
	$\log_8 14 - \log_8 3$
$\log_{13} \frac{h^2}{f}$	
	$\log_3 \pi - \log_3 5$
	$\log_{10} 1 - \log_{10} 7$
	$2 \log_{11} x + 6 \log_{11} x$
	$\log_{12} \frac{5}{3} + \log_{12} 4$

2. Rewrite each expression using the power law.
   a) $\frac{1}{2} \log_4 5$
   b) $\log_6 9^3$
   c) $\frac{1}{5} \log_8 18$
   d) $\log_2 7^{-5}$
   e) $\log_6 \sqrt{22}$
   f) $\log_9 \frac{1}{\sqrt{13}}$

3. Express as a single logarithm.
   a) $\log_3 5 + \log_3 8 + \log_3 15$
   b) $\log_4 8 - \log_4 10 + \log_4 3$
   c) $\log_2 19 + \log_2 4 - \log_2 31$
   d) $\frac{1}{2} \log 17 - \log 5$
   e) $\log (a + b) + \log (a^3)$
   f) $\log (x + y) - \log (x - y)$
   g) $4 \log x - 3 \log y$
   h) $\log_3 ab + \log_3 bc$

4. Rewrite each expression with no logarithms of products, quotients, or powers.
   a) $\log_7 (5x)$
   b) $\log_2 (m^3 n^2)$
   c) $\log_3 (abc)$
   d) $\log_9 (\sqrt[3]{y^2 + y})$
   e) $\log_8 \frac{\sqrt[4]{m}}{n}$
   f) $\log_6 (xy)^5$
   g) $\log_4 \frac{a^2 b}{\sqrt{c}}$
   h) $\log_3 \frac{1}{\sqrt{jk}}$

5. Evaluate.
   a) $\log_8 32 + \log_8 2$
   b) $\log_2 72 - \log_2 9$
   c) $\log_4 192 - \log_4 3$
   d) $\log_{12} 9 + \log_{12} 16$
   e) $\log_2 6 + \log_2 8 - \log_2 3$
   f) $\log_3 108 - \log_3 4$
   g) $\log_8 6 - \log_8 3 + \log_8 4$
   h) $\log_2 80 - \log_2 5$
   i) $\log 1.25 + \log 80$
   j) $\log_2 8^{27}$

6. Evaluate to four decimal places using a calculator.

a) $\log_5 12$ b) $\log_2 13$ c) $\log_7 9$
d) $\log_6 15$ e) $\log_9 8$ f) $\log_3 6$
g) $\log_4 7$ h) $\log_8 4$

Apply, Solve, Communicate

B **7.** Application Driving in fog at night greatly reduces the intensity of light from an approaching car. The relationship between the distance, d, in metres, that your car is from the approaching car and the intensity of light, $I(d)$, in lumens (lm), at distance d, is given by

$$d \doteq -166.67 \log \frac{I(d)}{125}$$

a) Solve the equation for $I(d)$.
b) How far away from you is an approaching car if $I(d) = 40$ lm?

8. Inquiry/Problem Solving Energy is needed to transport a substance from outside a living cell to inside the cell. This energy is measured in kilocalories per gram molecule, and is given by the relationship $E = 1.4 \log \frac{C_1}{C_2}$, where C_1 represents the concentration of the substance outside the cell, and C_2 represents the concentration inside the cell.

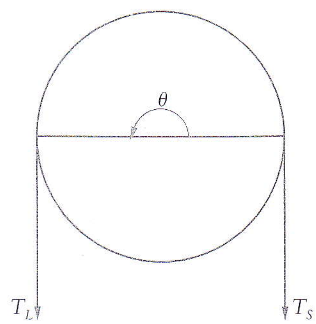
a) Find the energy needed to transport the exterior substance into the cell if the concentration of the substance inside the cell is
i) double the concentration outside the cell
ii) triple the concentration outside the cell
b) What is the sign of E if $C_1 < C_2$? Explain what this means in terms of the cell.

9. Communication Which is greater, $\log_6 7$ or $\log_8 9$? Explain.

10. The formula for the gain in voltage of an electronic device is $A_v = 20(\log V_o - \log V_i)$, where V_o is the output voltage and V_i is the input voltage.

a) Rewrite the formula as a single logarithm.
b) Verify the gain in voltage for $V_o = 22.8$ and $V_i = 14$ using both versions of the formula.

11. When a rope is wrapped around a fixed circular object, the relationship between the larger tension T_L and the smaller tension T_S is modelled by $0.434\mu\theta = \log \frac{T_L}{T_S}$, where m is the friction coefficient and θ is the wrap angle in radians.



a) Rewrite the formula using the laws of logarithms.
b) If the wrap angle is π (in radians), and a 200 N force is balancing a 250 N force, what is the friction coefficient?
c) If the rope is wrapped around the object 2.5 times, what force is now needed to balance the 250 N force?

C **12.** Show that if $\log_b a = c$ and $\log_y b = c$, then $\log_a y = c^{-2}$.

13. Use the product law of logarithms to prove the quotient law of logarithms,

$$\log_a \frac{p}{q} = \log_a p - \log_a q$$

where $a \in (1, \infty)$, $p, q \in (0, \infty)$.

14. Use the product and quotient laws of logarithms to prove the power law of logarithms $\log_a(p^c) = c \log_a p$, where $a \in (1, \infty)$, p, $q \in (0, \infty)$, $c \in (-\infty, \infty)$.

15. Derive the change of base formula,

$$\log_a x = \frac{\log_b x}{\log_b a}.$$

16. Find the error in the following calculation.

$$\log_3 0.1 < 2 \log_3 0.1$$
$$= \log_3 (0.1)^2$$
$$= \log_3 0.01$$
$$\log_3 0.1 < \log_3 0.01$$

Thus, $0.1 < 0.01$.

Key Concepts

- To solve exponential equations of the form found in Examples 1 and 2, first isolate the term containing the exponential variable on one side of the equation, then take the logarithm of each side of the equation, and apply the laws of logarithms to solve for the variable.
- To solve logarithmic equations of the form found in Examples 4 and 5, first isolate the terms with variables on one side of the equation, then use the laws of logarithms to express each side of the equation as a single logarithm, and simplify to solve for the variable.

Communicate Your Understanding

1. Describe two ways to verify the solution(s) to a logarithmic equation or an exponential equation.
2. Explain why logarithms are helpful in solving exponential equations.
3. Give an example of an exponential equation that cannot be solved exactly using the laws of logarithms. How would you solve the equation in this case?

Practise

Round your solutions to four decimal places, if necessary.

A 1. Determine if $x = 0.6$ is a root of $4^{2x} = 5$, and justify the result.

2. Solve for x and check your solution.
a) $\log x = 0$ b) $7^{4x-1} = 7$
c) $3^{x+3} = 9$ d) $\log_2 x = 8$
e) $\log_x 49 = 2$ f) $\log_{10} x = 3$
g) $\log_x 8 = \frac{3}{2}$ h) $\log_{10} 0.1 = x$

3. a) Solve each equation using the properties of logarithms.
b) Illustrate each solution graphically.
i) $\log_2 (x + 6) = 3$ ii) $\log_6 (x + 3) = 1$

4. Solve.
a) $5^x = 8$ b) $6^{3x} = 10$
c) $4^{\frac{x}{2}} = 7$ d) $3^{x+2} = 5$
e) $2^{6x-1} = 28$ f) $2^{4x} = 9^{x-1}$
g) $6(3)^{2x-3} = 18$ h) $2(7)^{4x-5} = 30$

5. Solve and check.
a) $4 \log_5 x = \log_5 625$
b) $-\log_3 1 = \log_3 7 - \log_3 x$
c) $\log_6 n = \frac{3}{4} \log_6 16$
d) $-\log_2 x - \log_2 3 = \log_2 12$
e) $\log 12 = \log 8 - \log x$
f) $\log 2^{3x} = \log 35$
g) $4 \log_6 x = \log_6 25$
h) $2 \log_7 x = \log_7 81$

6. Solve for x. Use a graphing calculator to verify your solution.
a) $\log x = -3$
b) $\log (x - 11) = 20$
c) $\log (4x - 1) = 39$
d) $\log_3 (5 - x) = 3$
e) $\log_2 (x + 6) + \log_2 3 = \log_2 30$
f) $\log_3 x + \log_3 (x - 1) = \log_3 (2x)$
g) $\log_6 (x + 3) + \log_6 (x - 2) = 1$
h) $\log_4 (x - 1) + \log_4 (x + 2) = 1$
i) $\log_2 (x + 1) - \log_2 (x - 1) = 1$
j) $1 - \log (x - 4) = \log (x + 5)$

7. Communication a) Show that $x = \log_5 4$ is a root of the equation $5^{2x} + 5^x - 20 = 0$.
b) Are there any other roots? Explain.

8. Solve for x and check your solution.
a) $2^{2x} - 2^x - 6 = 0$ b) $3^{2x} + 2(3^x) - 15 = 0$
c) $7^{2x} + 3(7)^x - 10 = 0$ d) $10^{2x} + 5(10)^x + 4 = 0$
e) $6^{2x} - 2(6)^x - 15 = 0$ f) $4^{2x} + 9(4)^x + 14 = 0$

Apply, Solve, Communicate

E **9.** The function $A = P(1.06)^n$ represents the amount, A, in dollars, in an investment n years from now, where P is the original investment, or principal, in dollars. If Seth invests \$1000 now, find how long it will take to accumulate to

a) \$1226.23 b) \$1664.08

c) double his original investment

d) \$5000.00

10. Application The intensity of light, I, in lumens, passing through a certain type of glass is given by $I(t) = I_0(0.97)^x$, where I_0 is the initial intensity and x is the thickness of the glass, in centimetres.

a) What thickness of the glass will reduce the intensity of light to half its initial value?

b) What effect does doubling the thickness of the glass have on the intensity of light passing through it?

11. Application The average annual salary, S, in dollars, of employees at a particular job in a manufacturing company is modelled by the equation $S = 25\,000(1.05)^n$, where \$25 000 is the initial salary, which increases at 5% per year.

a) How long will it take the salary to increase by 50%?

b) If the starting salary is \$35 000, how long will it take the salary to increase by 50%? Explain your answer.

12. How long, to the nearest month, will it take for an investment of \$600 at 5.5%, compounded annually, to

a) double? b) triple?

c) accumulate to \$900?

13. Inquiry/Problem Solving The speed, v, in kilometres per hour, of a water skier who drops the towrope, can be given by the formula $v = v_0\,(10)^{-0.23t}$, where v_0 is the skier's speed at the time she drops the rope, and t is the time, in seconds, after she drops the rope. If the skier drops the rope when travelling at a speed of 65 km/h, how long will it take her to slow to a speed of 13 km/h?

14. Inquiry/Problem Solving On average, the number of items, N, per day, on an assembly line, that a quality assurance trainee can inspect is $N = 40 - 24(0.74)^t$, where t is the number of days worked.

a) After how many days of training will the employee be able to inspect 32 items?

b) The company expects an experienced quality assurance employee to inspect 45 items per day. After the training period of 15 days is complete, how close will the trainee be to the experienced employee's quota?

15. The number of hours, $H(t)$, that cheese will remain safe to eat decreases exponentially as the temperature of the surrounding air, t, in degrees Celsius, increases. For a particular type of cheese, this relationship is represented by $H(t) = 140(10)^{-0.034t}$. To the nearest hour, how long will the cheese remain safe to eat if it is stored at

a) 0°C? b) 16°C? c) 25°C?

16. Use the formula from Example 6 (page 439) to determine the altitude of a rock climber, if the atmospheric pressure is approximately 95 kPa.

17. The designers of aircraft must know the external pressure in order to control the pressure inside the aircraft. What range of external pressure must be controlled for a small airplane with a maximum altitude of 10 km?

18. The intensity of the sunlight below the surface of a large body of water is reduced by 4.6% for every metre below the surface. Show that the depth at which the sunlight has intensity $I(d)$, in lumens, is given by $d \doteq -48.9 \log \dfrac{I(d)}{I_0}$, where I_0 is the initial intensity and d is the depth, in metres.

19. Solve for x and check your solution.

a) $\log_2 x + \log_4 x + \log_8 x + \log_{16} x = 25$

b) $2(5^{6x}) - 9(5^{4x}) + 13(5^{2x}) - 6 = 0$

Region, Northwest Territories, an earthquake of magnitude 6.9 occurred. On Vancouver Island, on June 23, 1946, an earthquake about 2.5 times as intense occurred. Was the Vancouver Island earthquake strong enough to cause metal buildings to collapse?

5. The absolute magnitude of a star, M, is related to its luminosity, L, by the formula $M = 4.72 - \log \frac{L}{L_0}$, where L_0 is the luminosity of the sun. The luminosity is the rate at which the star emits light, and is measured in watts.
a) Determine the absolute magnitude of the sun.
b) The absolute magnitude of Sirius, the star that, other than the sun, appears brightest from Earth, is 1.41. Is Sirius more or less luminous than the sun? By what factor?
c) Repeat part b) for Canopus, the star that appears second brightest from Earth, which has an absolute magnitude of −4.7.
d) A distant object called a quasar has a luminosity of about 10^{38} W. The sun's luminosity is about 4×10^{26} W. Determine the absolute magnitude of the quasar.

Web Connection
For more on solar eclipses, and some spectacular photos, go to **www.mcgrawhill.ca/links/CAF12** and follow the links.

MODELLING MATH

6. Communication An airplane altimeter is a gauge that indicates the height of the plane above ground. It works based on air pressure, according to the formula $h = 18\,400 \log \frac{P_0}{P}$, where h is the height of the airplane above the ground, in metres, P is the air pressure at height h, and P_0 is the air pressure at ground level. Air pressure is measured in kilopascals.

The height of aircraft is still usually measured in feet, not metres.

a) Air pressure at the ground is 102 kPa. If the air pressure outside the airplane is 32.5 kPa, what is the height of the airplane?
b) How high would the airplane have to be flying for the outside air pressure at that height to be half of the air pressure at ground level?
c) If the weather changes, then the air pressure at ground level may change. How do pilots take this into account?

MODELLING MATH

7. Communication a) Estimate the typical air pressure at the peak of Mount Everest. Use the formula from question 6.
b) Do research to determine whether your estimate in part a) is reasonable.
c) Use the results of part a) to explain why climbers of tall mountains often use oxygen tanks to help them breathe.

8. Inquiry/Problem Solving Another logarithmic scale is used for welding glasses, which protect the eyes from bright light. The shade number of welding glasses is given by the equation

$$\text{shade \#} = \frac{7(-\log_{10} T)}{3} + 1,$$ where T is the fraction of visible light that the glass transmits.

a) When there is a solar eclipse, it is safe to look at it through #14 welding glasses. What fraction of visible light is transmitted by #14 welding glasses?
b) A furnace repair person should wear #2 welding glasses. What fraction of visible light is transmitted by #2 welding glasses?
c) How many times as much visible light is transmitted by #2 welding glasses as by #14 welding glasses?

9. For electric welding, a safe fraction of visible light is 5.1795×10^{-5}. What shade number of welding glasses, to the nearest unit, should an electric welder use?

10. Application Johannes Kepler (1571–1630) discovered a relationship between the average distance of a planet to the sun, in millions of kilometres, and the time, in days, it takes the