Composition of Functions

Dec 21/2016

Suppose you were asked to graph $y = 2^{\sin x}$.

To make matters worse, suppose your calculator could only perform one operation at a time (i.e., you could perform the exponential operation, or the sine operation, but not both).

How would you get the points for your graph?

x	$\sin x$		x_2	2^{x_2}
0	$\sin(0) = 0$	$\longrightarrow$	0	$2^0 = 1$
1	$\sin(1) \doteq 0.8415$	$\longrightarrow$		$2^{0.8415} \doteq 1.79$
2	$\sin(2) \doteq 0.9093$	$\longrightarrow$		$2^{0.9093} \doteq 1.88$
3	$\sin(3) =$			

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Composition of Functions

One way to view a function is as a machine, with an input (the independent variable, x) and an output (the dependent variable, y).

$$x \longrightarrow \boxed{f(x)} \longrightarrow y$$

input machine output

It is possible to connect multiple functions (machines) together, so the output of the first is the input to the second.

$$x \longrightarrow f(x) \longrightarrow y \longrightarrow g(y) \longrightarrow z$$

1st input

1st output
is
2nd input

2nd output

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A composition of functions occurs when the argument of a function is another function.

$$(f \circ g)(x) = f(g(x))$$

f composed with g
"f of g of x"

outer function
(calculate 2nd)

inner function
(calculate 1st)

Ex.1 Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$

(a) $(f \circ g)(x)$

$= f(g(x))$

$= f(x^2 - 4)$

$= \sqrt{x^2 - 4}$

(b) $(g \circ f)(x)$

$= g(f(x))$

$= g(\sqrt{x})$

$= (\sqrt{x})^2 - 4$

$= x - 4$, $x \geq 0$

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Ex.1 Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$

(c) create a table of values for $(f \circ g)(x)$

(d) determine the domain of $f \circ g$

x	$g(x) = x^2 - 4$	x	$f(x) = \sqrt{x}$
-3	$(-3)^2 - 4 = 5$	5	$f(5) = \sqrt{5}$
-2	$(-2)^2 - 4 = 0$	0	$f(0) = \sqrt{0} = 0$
-1	$(-1)^2 - 4 = -3$	-3	$f(-3) = \sqrt{-3}$ undef.
0	$= -4$		$\sqrt{-4}$ undef
1	$= -3$		$\sqrt{-3}$ undef
2	$= 0$		$\sqrt{0} = 0$
3	$= 5$		$\sqrt{5}$

$D_{f \circ g} = \{x \in \mathbb{R} \mid x \leq -2, x \geq 2\}$

OR

$D_{f \circ g} = \{x \in \mathbb{R} \mid |x| \geq 2\}$

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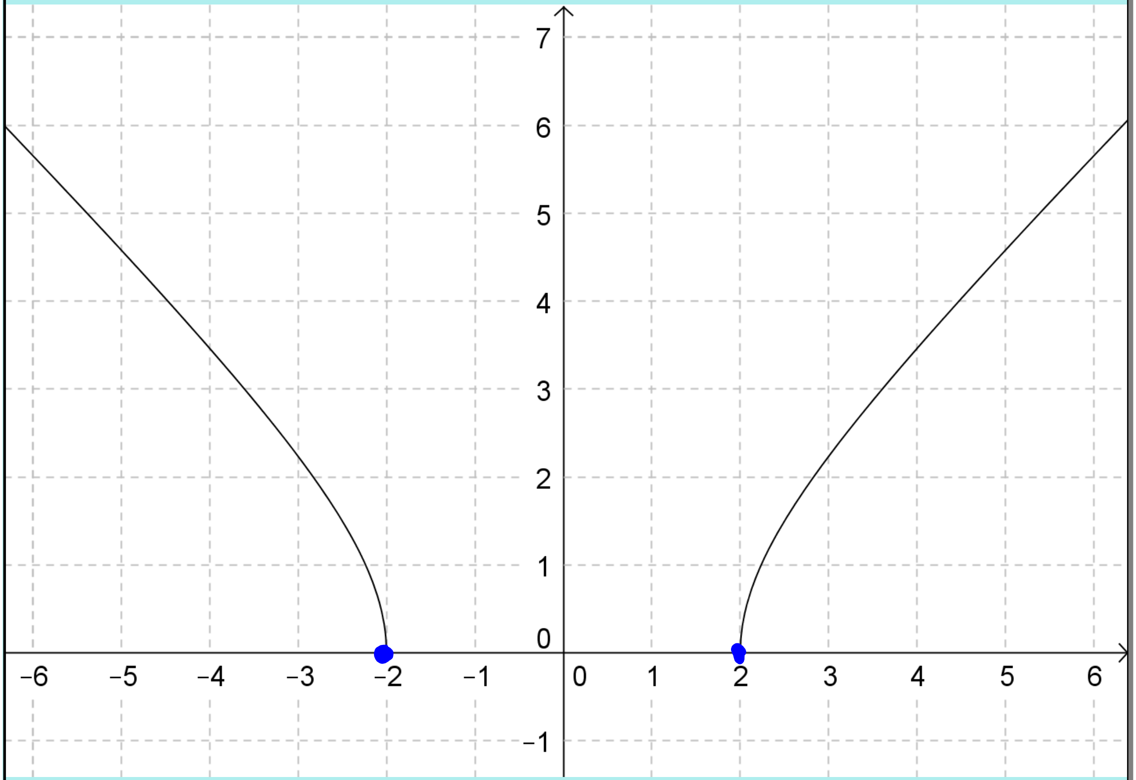
$$(f \circ g)(x) = \sqrt{x^2 - 4}$$



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Ex.1 Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$

(e) determine the domain algebraically. outer $\rightarrow f \circ g \leftarrow$ inner

When determining the domain of $f \circ g$ algebraically:
(1) determine the domain (restrictions) of the outer function.
(2) create an equation or inequality using the inner function.
(3) solve for the restrictions on the inner function.

$f(g(x))$

① outer: $f(x)$

$D_f = \{x \in \mathbb{R} \mid x \geq 0\}$

② $g(x) \geq 0$

$x^2 - 4 \geq 0$

③ $(x-2)(x+2) \geq 0$

	−3	−2	0	2	3
$x-2$	−		−		+
$x+2$	−		+		+
	+		−		+
	pass		fail		pass

$D_{f \circ g} = \{x \in \mathbb{R} \mid x \leq -2, x \geq 2\}$

OR

$D_{f \circ g} = \{x \in \mathbb{R} \mid |x| \geq 2\}$

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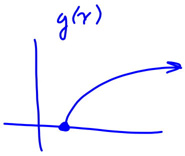
Assigned Work:

p.552 # 1, 2abf, 3, 5aef, 6def, 7cf, 10, 13

6(d) $f(x) = 2^x$ $\quad g(x) = \sqrt{x-1}$

$$f \circ g = f(g(x)) = 2^{\sqrt{x-1}}$$

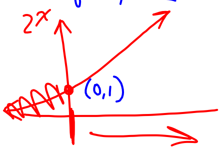


① $D_f = \{x \in \mathbb{R}\}$

② $D_g = \{x \in \mathbb{R} \mid x \geq 1\}$

$R_g = \{y \in \mathbb{R} \mid y \geq 0\}$

③ $f(g(x)) = 2^{\sqrt{x-1}}$ only positive and zero



$R_{f \circ g} = \{y \in \mathbb{R} \mid y \geq 1\}$

$D_{f \circ g} = \{x \in \mathbb{R} \mid x \geq 1\}$

$$x \longrightarrow g(x_1) \longrightarrow f(x_2) \longrightarrow y$$

$\sqrt{x-1}$ $\quad 2^x$

$D_{f \circ g}$ $\quad R_{f \circ g}$

$x_1 \geq 1$ $\quad x_2 \geq 0$ $\quad y \geq 1$

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7(f) $h(x) = \sqrt[3]{(x+4)^2}$

$$f(g(x)) = \sqrt[3]{(x+4)^2}$$

$g(x) = (x+4)^2$

$f(x) = \sqrt[3]{x}$

$g(x) = x + 4$ $\quad f(x) = x^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2$

$$= \sqrt[3]{x^2}$$

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10. $C(n) = 975 + 39.95n$

next month : $D(c) = 0.80\, C$

$$D(C(n)) = 0.80(975 + 39.95n)$$

$$D(n) =$$

discounted cost in terms of n

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13. $v(t) = 40 + 3t + t^2$

$$c(v) = \left(\frac{v}{500} - 0.1\right)^2 + 0.15$$

$$c(v(t)) = \left(\frac{40 + 3t + t^2}{500} - 0.1\right)^2 + 0.15$$

gas consumption

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