Review – Solutions

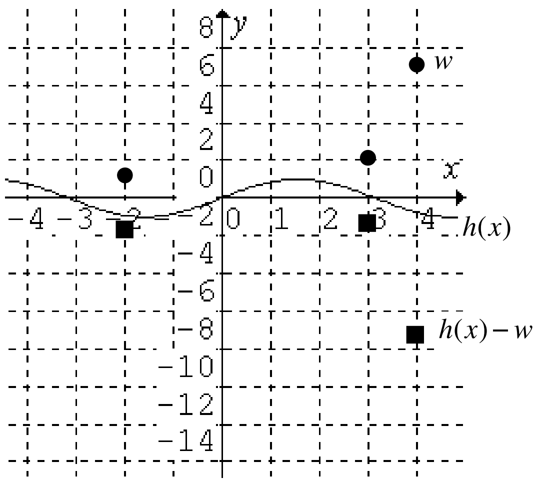
Use the following functions to answer questions 1 – 5

$f(x) = x^2 - 1$, $g(x) = -2^x$, $h(x) = \sin x$, $w = \{(-2,\ 1), (3,\ 2), (4,\ 7)\}$, $r(x) = \frac{x}{x+1}$, and $l(x) = 2\log x$,

1) Determine:

a) $f + w$ first determine the corresponding points in $f(x)$ $f(-2) = (-2)^2 - 1 = 3$ $\quad f(3) = 3^2 - 1 = 8$ $\quad f(4) = 4^2 - 1 = 15$ Then add the y values. $f + w = \{(-2, 4), (3, 10), (4, 22)\}$	b) $f \cdot r$ $= (x^2 - 1) \cdot \frac{x}{x+1}$ $= \cancel{(x+1)}(x-1) \cdot \frac{x}{\cancel{x+1}}$ $= x(x-1), \quad x \neq -1$
c) $\frac{h}{l} = \frac{\sin x}{2\log x}$	d) $h \circ f$ $= h(f(x))$ $= \sin(x^2 - 1)$
e) $g \circ w$ substitute the y values in w, as x, into $g(x)$ to determine the y values of the composition. $g(w(-2)) = g(1) \Rightarrow -(2^1)$ $g(w(3)) = g(2) \Rightarrow -(2^2)$ $g(w(4)) = g(7) \Rightarrow -2^7$ $g \circ w = \{(-2, -2), (3, -4), (4, -128)\}$	f) $w \circ r$ since the y values of r will become the x values of w, we need to find the values of x in r that will result in the correct y values. $\frac{x}{x+1} = -2$ $\quad \frac{x}{x+1} = 3$ $\frac{x}{x+1} + 2 = 0$ $\quad \frac{x}{x+1} - 3 = 0$ $\frac{x + 2(x+1)}{x+1} = 0$ $\quad \frac{x - 3(x+1)}{x+1} = 0$ $x + 2x + 2 = 0, x \neq -1$ $\quad x - 3x - 3 = 0, x \neq -1$ $3x = -2$ $\quad -2x = 3$ $x = \frac{-2}{3}$ $\quad x = \frac{-3}{2}$ $\frac{x}{x+1} = 4$ $\frac{x}{x+1} - 4 = 0$ $\frac{x - 4(x+1)}{x+1} = 0$ $x - 4x - 4 = 0, \ x \neq -1$ $-3x = 4$ $x = \frac{-4}{3}$ $w \circ r = \{(\frac{-2}{3}, 1), (\frac{-3}{2}, 2), (\frac{-4}{3}, 7)\}$

2) Graph $h(x)$ and w. Use your graphs to graph $h(x)-w$.



3) Determine:

a) $D_{g\cdot l}$

$D_{g\cdot \ell} = D_g \cap D_\ell$

$= \{x \mid x > 0, x \in \mathbb{R}\}$

rough work

$D_g = \{x \mid x \in \mathbb{R}\}$

$D_\ell = \{x \mid x > 0, x \in \mathbb{R}\}$

b) $D_{l(r(x))}$

$\ell(r(x)) = 2\log\left(\frac{x}{x+1}\right)$

$\frac{x}{x+1} > 0$

zeros $x = 0$

VA $x = -1$

+ − +

−1 0

✓ ✗ ✓

$D_{l(r(x))} = \{x \mid x < -1 \text{ or } x > 0, x \in \Re\}$

c) $R_{h(r(x))}$

$R_h = \{y \mid -1 \le y \le 1, y \in \mathbb{R}\}$

$R_r = \{y \mid y \ne 1, y \in \mathbb{R}\}$

$\therefore x \ne 1$ in $h(x)$

$\therefore \sin 1 \ne h(1)$

but $\sin 1 = \sin(\pi - 1)$

$\therefore$ y value still exists

$R_{h(r(x))} = \{y \mid -1 \le y \le 1, y \in \Re\}$

d) $R_{w(l(x))}$

$R_w = \{1, 2, 7\}$

$R_\ell = \{y \mid y \in \mathbb{R}\}$

$\therefore$ no extra restrictions on R_w

$R_{w(l(x))} = \{1, 2, 7\}$

4) How many zeros does $r(x)\cdot f(x)$ have?

$r(x)$ has one zero; $x = 0$

$\ell(x)$ has one zero; $x = 1$

Thus the product will have two zeros: $x = 0$ and $x = 1$

5) Will the product of $f(x)$ and $h(x)$ be even, odd, or neither?

$f(x)$ is an even function and $h(x)$ is an odd function thus the product will be an odd function.

$f(x)$ is an even fnct proof: $f(x) = x^2 - 1$

$$f(-x) = (-x)^2 - 1$$
$$= x^2 - 1$$
$$= f(x)$$

$h(x)$ is an odd fnct proof $h(x) = \sin x$

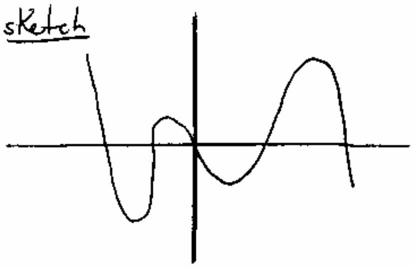
$$h(-x) = \sin(-x)$$
$$= -\sin x$$
$$= -h(x)$$

if $x \in QI$
$-x \in QIV$

The product of an even and an odd fnct is odd

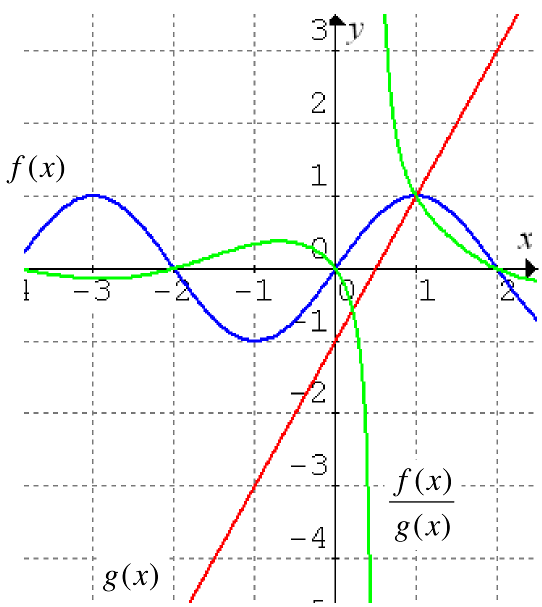
$\therefore$ the product of $f(x)$ and $h(x)$ is odd

sketch



Window
$x_{min} = -2\pi$
$x_{max} = 2\pi$
$x_{scl} = \pi/2$
$y_{min} = -4$
$y_{max} = 4$
$y_{scl} = 1$

6) Graphs of $y = f(x)$ and $y = g(x)$ are given below. Sketch $h(x) = \dfrac{f(x)}{g(x)}$ on the grid above.

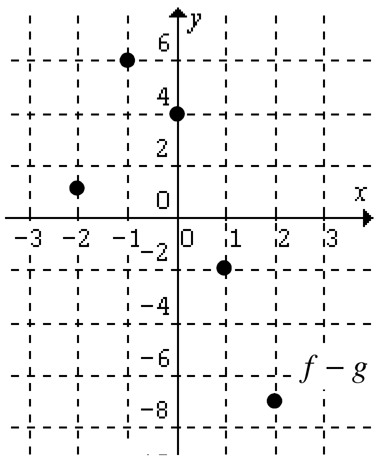


7) Given $f = \{(-2, 6), (-1, 8), (0, 5), (1, 0), (2, -2)\}$ and $g(x) = x^2 + 1$,

a) graph $f - g$

In order to subtract these functions we need to find the points on g(x) that match the *x* values of the points on *f(x)*

matching pts: $(-2, 5), (-1, 2), (0, 1) \quad (1, 2), (2, 5)$



b) state the domain of $f \cdot g$: $D_{f \cdot g} = \{-2, -1, 0, 1, 2\}$

$D_f = \{-2, -1, 0, 1, 2\} \quad D_g = \{x \mid x \in \mathbb{R}\}$

$D_{f \cdot g} = D_f \cap D_g$

(The overlap of the domains)

c) determine $f \circ g(x)$ when $x = -1$

$f \circ g = f(g(x))$

$f(g(-1)) = f(2)$
$= -2$

d) determine $f(g(-1))$

same as c) … I meant $g(f(-1))$

$g(f(-1)) = g(8)$
$= 65$

e) determine the domain of $f(g(x))$

values of x that make $R_g = D_f$: $x^2 + 1 = 1$, $x^2 + 1 = 2$

The other 3 *x* values are not possible since $x^2 + 1 \geq 1$ for all *x* values.

$D_{f \circ g} = \{0, \pm 1\}$

8) If $f(x)=\dfrac{x}{x-1}$ and $g(x)=\dfrac{3}{x^2-1}$, find the value of x for which $(f+g)(x)=1$.

$(f+g)(x) = \dfrac{x}{x-1} + \dfrac{3}{x^2-1}$

$(f+g)(x) = \dfrac{x}{x-1} + \dfrac{3}{(x+1)(x-1)}$

$(f+g)(x) = \dfrac{x(x+1)+3}{(x+1)(x-1)}$

$1 = \dfrac{x^2+x+3}{(x+1)(x-1)}$

$(x+1)(x-1) = x^2+x+3 \quad x \neq \pm 1$

$x^2-1 = x^2+x+3$

$0 = x^2+x+3-x^2+1$

$0 = x+4$

$-4 = x$

9) Given $f(x)=x+3$, determine

a) $f^{-1}(x)$ b) $f \circ f^{-1}$ c) $f^{-1} \circ f$

a) $f^{-1}(x) \rightarrow$ inverse

f: $y = x+3$

f^{-1}: $x = y+3$

$x-3 = y$, a fnct

$\therefore$ $x-3 = f^{-1}(x)$

b) $f \circ f^{-1}$

$= f(f^{-1}(x))$

$= f(x-3)$

$= x-3+3$

$= x$

c) $f^{-1} \circ f$

$= f^{-1}(f(x))$

$= f^{-1}(x+3)$

$= x+3-3$

$= x$

as stated in Thw of composition: $f \circ f^{-1} = f^{-1} \circ f = x$

as long as $D_f = D_{f^{-1}}$

a) $f^{-1}(x)=x-3$ b) $f \circ f^{-1}=x$ c) $f^{-1} \circ f = x$

10) Use composition to verify that $f(x)=\dfrac{1}{x+1}$ and $g(x)=\dfrac{1-x}{x}$ are inverses of each other.

If f and g are inverses then $f \circ g = x$

$f \circ g = f(g(x))$

$= \dfrac{1}{\frac{1-x}{x}+1}$

$= 1 \div \left(\frac{1-x}{x}+1\right)$

$= 1 \div \left(\dfrac{1-x+x}{x}\right)$

$= 1 \div \dfrac{1}{x}$

$= 1 \cdot \dfrac{x}{1}$

$= x$ $\therefore$ f and g are inverse fncts.

11) Given $f(x) = \sin x$ and $g(x) = 2x - \frac{\pi}{3}$, describe the graph of $f \circ g$ as a transformation of the graph of *f*.

$$f \circ g = \sin\left(2x - \frac{\pi}{3}\right)$$
$$= \sin 2\left(x - \frac{\pi}{6}\right)$$

f(x) has been compressed horizontally by a factor of 2
translated to the right by $\frac{\pi}{6}$ units

12) Given $q(x) = \frac{2x-1}{x+5}$,

a) determine the intercept(s), asymptote(s), intervals of increase/decrease, and end behaviour of the function,

a) x-int: let $q(x) = 0$
$\Rightarrow 2x - 1 = 0$
$2x = 1$
$x = \frac{1}{2}$

y-int let $x = 0$
$q(0) = \frac{-1}{5}$

VA let denominator $= 0$
$x + 5 = 0$
$x = -5$

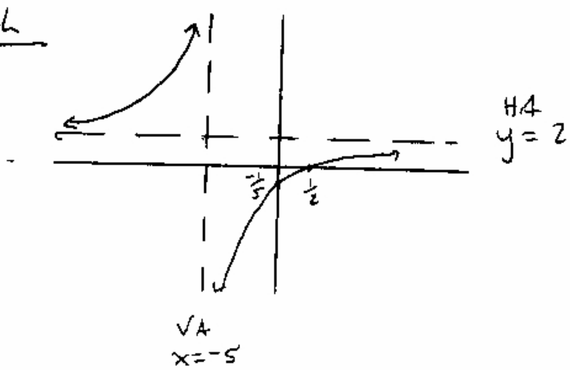
HA: deg of numerator = deg of denominato
$\therefore$ HA $y = 2$
(divide lead. coeff.)

$2x - 1$ is incr. $\forall x \in \mathbb{R}$
$x + 5$ is incr. $\forall x \in \mathbb{R}$
$\therefore q(x)$ is incr. $\forall x \in \mathbb{R}$

as $x \to \infty$, $y \to 2$
as $x \to -\infty$, $y \to 2$

b) sketch the graph of the function,

sketch



c) define an equation $f(x)$ and an equation $g(x)$ such that $q(x)=\frac{f(x)}{g(x)}$

$f(x)=2x-1$

$g(x)= x+5$

d) justify the properties you found in a) by studying the properties of the functions $f(x)$ and $g(x)$.
$D_{q(x)}=D_f \cap D_g$ *and* $g(x)\neq 0$; number of zeros = number of zeros of *f*, if in domain.

d) $D_{q(x)}$ should be $D_f \cap D_g$ and $g(x) \neq 0$

$D_f = \{x \mid x \in \mathbb{R}\}$ $D_g = \{x \mid x \in \mathbb{R}\}$ $x+5 \neq 0$

$x \neq 5$ VA!

intervals of incr. have already been explained

# of x-int: at most # zeros of numerator (f(x))

f(x) has 1 zero, $x = 0.5$ and $0.5 \in D_{q(x)}$

$\therefore$ q(x) has one x-int.

both functions have neither odd nor even symmetry and so their quotient also has neither symmetry.

13) Given $f(x)=\sec x$, where , $-2\pi \leq x \leq 2\pi$ and $g(x)=\log x$ determine,

a) the domain of $\frac{f}{g}$

a) $\frac{f}{g} = \frac{\sec x}{\log x} = \frac{1}{(\log x)(\cos x)}$

$D_f: \{x \mid x \neq \frac{\pi}{2} + n\pi, -2\pi \leq x \leq 2\pi\}$

$D_g: \{x \mid x > 0, x \in \mathbb{R}\}$ $g(1) = 0$

$\frac{f}{g}$ has VA at:

the VA of *f(x)* [$x = \pi/2, 3\pi/2$] and

the zeros of *g(x)* [$x = 1$]

$\frac{f}{g}$ has a hole at the VA of *g(x)* [$x = 0$]

$$D_{\frac{f}{g}} = \left\{ x \middle| x \neq 1, x \neq \frac{\pi}{2}, \frac{3\pi}{2}, 0 < x \leq 2\pi, x \in \Re \right\}$$

b) at most, the number of zeros for $\frac{f}{g}$, do you think this number is accurate?

b) # zeros of f(x) = none

zeros of g(x) = 1 ($x = 1$)

at most $\frac{f}{g}$ could have # zeros of f(x)

$\therefore$ # zeros of $\frac{f}{g}$ = none, accurate!

c) the domain of $\frac{g}{f}$

$$\frac{g}{f} = \frac{\log x}{\sec x} \Rightarrow \frac{\log x}{1/\cos x}$$

$$= (\log x)(\cos x)$$

but $\cos x \neq 0$

$D_g : \{x \mid x > 0, x \in \mathbb{R}\}$

$D_f : \{x \mid -2\pi \le x \le 2\pi, x \neq \frac{\pi}{2} + n\pi\}$

$f(x) = 0$ has no sol$^{\underline{n}}$

$D_{\frac{g}{f}} : D_g \cap D_f$ and sol$^{\underline{n}}$ to $f(x) \neq 0$

$: \{x \mid 0 < x \le 2\pi, x \neq \frac{\pi}{2} + n\pi, x \in \mathbb{R}, n \in \mathbb{Z}\}$

$$D_{\frac{g}{f}} = \left\{ x \middle| x \neq \frac{\pi}{2}, \frac{3\pi}{2}, 0 < x \le 2\pi, \ x \in \Re \right\}$$

$\frac{g}{f}$ has VA at:

the VA of $g(x)$ [$x = 0$] and

the zeros of $f(x)$ [none]

$\frac{g}{f}$ has a hole at the VA of $f(x)$ [$x = \pi/2, 3\pi/2$]

d) at most, the number of zeros for $\frac{g}{f}$, do you think this number is accurate?

\# zeros of $g(x) = 1$ $(x = 1)$

\# zeros of $f(x)$ = none

at most $\frac{g}{f}$ could have \# zeros of $g(x)$

$\therefore$ at most, \# zeros of $\frac{g}{f} = 1$

zero of $g(x) \in D_{\frac{g}{f}}$ $\therefore$ This is accurate

note: from the graph is looks like the function has zeros at the VA of f(x), but they are actually holes

14) Given $f(x) = 3^x$ and $g(x) = \tan x$, where $-2\pi \le x \le 2\pi$ determine,

a) the domain of $f \cdot g$

$D_{f \cdot g} = D_f \cap D_g$

"overlap"

"common values"

$D_f : \{x \mid x \in \mathbb{R}\}$

$D_g : \{x \mid -2\pi \le x \le 2\pi, x \neq \frac{\pi}{2} + n\pi, x \in \mathbb{R}\}$

$$D_{f \cdot g} = \left\{ x \middle| x \neq \frac{\pi}{2} + n\pi, -2\pi \le x \le 2\pi, n \in \mathrm{Z}, x \in \Re \right\}$$

b) the range of $f \cdot g$

$R_{f \cdot g}$ at most $R_f \cdot R_g$

$R_{f \cdot g}: \{y \mid y \in \mathbb{R}\}$

$R_f: \{y \mid y > 0, y \in \mathbb{R}\}$

$R_g: \{y \mid y \in \mathbb{R}\}$

Any real number multiplied by a number larger than zero will still be a real number.

c) at most, the number of zeros for $f \cdot g$, do you think this number is accurate?

zeros of 3^x is 0

zeros of $\tan x$ (within $-2\pi \le x \le 2\pi$) is 5 ($x = \pm 2\pi, \pm\pi, 0$)

The # of zeros of $f \cdot g = 0 + 5$ at most
$= 5$

Since the zeros are within $D_{f \cdot g}$ This number is accurate

At most, the number of zeros of $f \cdot g$ = number of zeros of f + number of zeros of g.
(Assuming there are no restrictions on the domain or common zeros, i.e.: double roots).
Thus, at most, the number of zeros of $f \cdot g$ = 5.
All the zeros are within the domain and there are no common (double) zeros, thus this number is accurate.

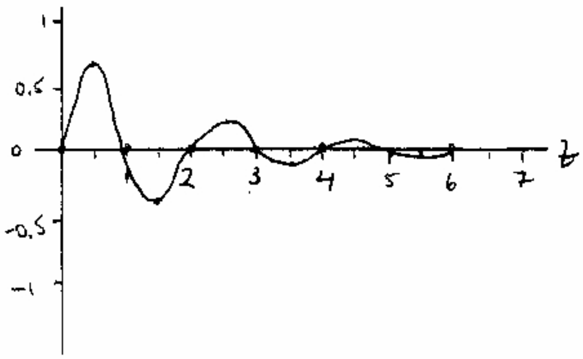
15) The plucked string of a guitar and the sound as it fades away can be represented by a damped sine wave that has an equation of the form $y = 2^{-t} \sin \pi t$. Sketch a graph of the functions for $0 \le t \le 2\pi$.

2^{-t}: exponential decreasing $\quad (2^{-1})^t = \left(\frac{1}{2}\right)^t$

$\therefore$ decreasing by 50%

$\sin \pi t$: trigonometric, amplitude of 1

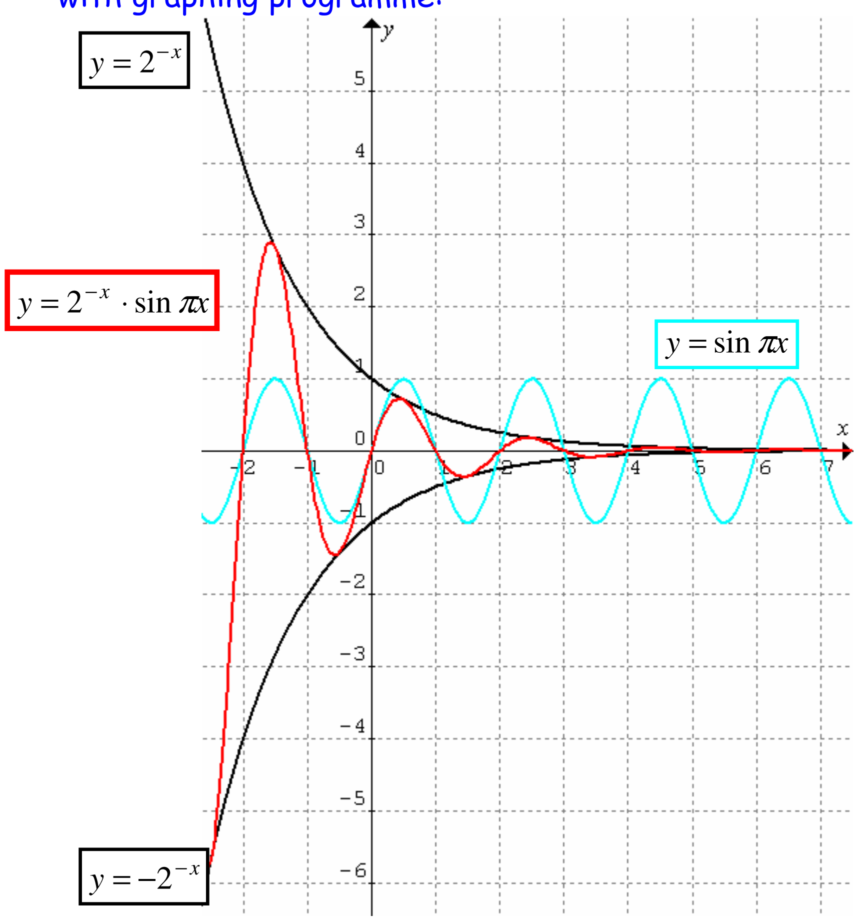
period $= \frac{2\pi}{\pi}$



t	y
0	$2^{-0} \sin 0 = 0$
0.5	$2^{-0.5} \sin(0.5\pi) \approx 0.7071$
1	$2^{-1} \sin \pi = 0$
1.5	$2^{-1.5} \sin(1.5\pi) \approx -0.3536$
2	$2^{-2} \sin 2\pi = 0$
2.5	$2^{-2.5} \sin(2.5\pi) \approx 0.1768$
3	$2^{-3} \sin 3\pi = 0$
3.5	$2^{-3.5} \sin(3.5\pi) \approx -0.0884$

etc.

with graphing programme:



16) Let S(t) represent the number of single adults in Canada in year t and M(t) represent the number of married adults in Canada in year t. Let E(t) represent the average amount spent on entertainment by a single adult and let N(t) represent the average amount spent on entertainment by a married adult. Using a combination of the functions defined above come up with representations for the following functions:

a) A(t), the number of Canadian adults in Canada in year t.

A(t) = S(t) + M(t), assuming that all adults are either single or married.

b) B(t), the amount of money spent on entertainment by Canadian single adults in year t.

B(t) = (amount spent per single adult) * (number of single adults)

*B(t) = E(t) * S(t)*

c) C(t), the amount of money spent on entertainment by Canadian adults in year t.

C(t) = amount spent by single adults + amount spent by married adults.

*C(t) = B(t) + M(t) * N(t)*

*C(t) = E(t) * S(t) + M(t)*N(t)*

17) The *change function* is defined as $d(x) = f(x) - f(x-1)$.

change fnct. $d(x) = f(x) - f(x-1)$

$f(x) \rightarrow y$ value

$f(x-1) \rightarrow y$ value for a point to the left of $f(x)$

What is the meaning, in terms of *f*, if

a) $d(x) > 0$

$f(x) - f(x-1) > 0$

$f(x) > f(x-1)$

$\therefore f(x)$ is increasing

y values are getting bigger.

f is increasing

b) $d(x) < 0$

$f(x) - f(x-1) < 0$

$f(x) < f(x-1)$

$\therefore f(x)$ is decreasing

y values are getting smaller.

f is decreasing

c) $d(x) = 0$

$f(x) - f(x-1) = 0$

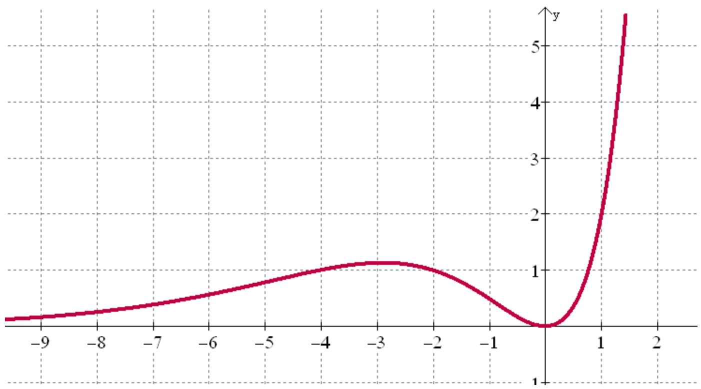
$f(x) = f(x-1)$

$\therefore f(x)$ is constant

$\rightarrow$ horizontal line

y values are all equal.

18) Each of the following graphs is a combination of two of the functions: $f(x)=x$, $g(x)=2^x$, $h(x)=x^2$, $j(x)=\cos x$ and one of the operations: addition, subtraction, multiplication, division, for each of the given graphs. State the equation and explain how you know using key features of the functions.

<table>
<tr><th>Graph</th><th>Equation and Explanation</th></tr>
<tr><td>
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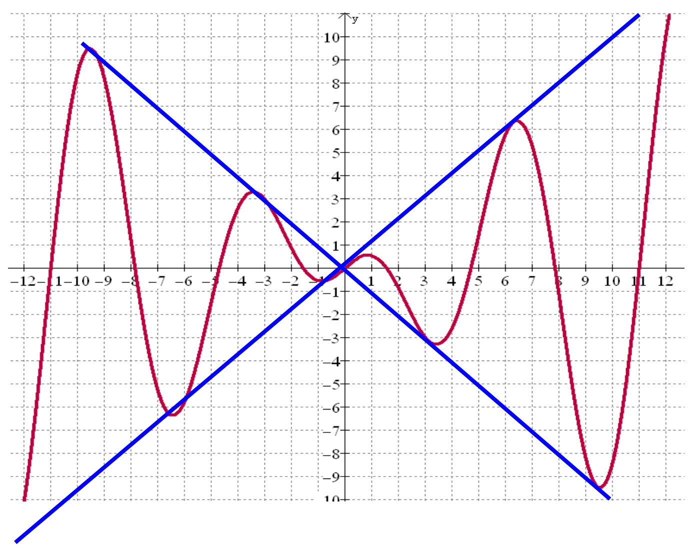
$y = h(x) \cdot g(x)$

$y = x^2 \cdot 2^x$

The general shape of the curve is that of an exponential, thus it must be a combination with *g(x)*.
The function has a zero at *x = 0*. *g(0) = 1*, thus either 1 was subtracted or the function was multiplied by a function with a zero at *x = 0*. (not divided because no asymptotes/holes).
The *y* values of a cosine oscillate between -1 and 1 thus the difference between *g(x)* and *f(x)* would also oscillate.
Hence the function must be a product. The zero is a turning point thus the function is a product of *g(x)* and a function with a double root, *h(x)*.</td></tr>
<tr><td>
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$y = f(x) \cdot j(x)$

$y = x \cdot \cos x$

The general shape of the curve is that of a sinusoidal, thus it must be a combination with *j(x)*.
The function has a zero at *x = 0*. *j(0) = 1*, thus either 1 was subtracted or the function was multiplied by a function with a zero at *x = 0*. (not divided because no asymptotes/holes).
The *y* values of a cosine oscillate between -1 and 1 now they seem to be oscillating between a linear function (when you join one set of extrema they form the line y = x; the other set forms the line y = - x. The function is very likely a product of *j(x)* and *f(x)* .
Further supporting evidence:
The zero at x = 0 is a single zero. The zeros of the function are those of both *j(x)* and *f(x)*
j(x) is an even function and *f(x)* is an odd function; the product of an even and an odd functions is odd, such as this function.</td></tr>
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19) Suppose that some oil has been spilled in water and has formed a circular oil slick. One minute after the spill the radius of the slick is 2 metres and 3 minutes after the spill the radius is 6 metres.

a) Express the radius, *r*, of the spill as a function of time, *t*, if the radius is increasing at a constant rate.

$t = 1$ min $\quad r = 2$m

$t = 3$ min $\quad r = 6$m

rate is constant $\therefore$ avg. rate $= \frac{6-2}{3-1}$

$m = 2$

$r = 2t + b$, let b rep the y-int.

$6 = 2(3) + b$ find b by subing in pt.

$6 = 6 + b$

$0 = b$

$r(t) = 2t$ is the eq'n of radius as a fnct of time

b) Was the radius 0 at time $t = 0$?

$r(0) = 0$

c) Express the circumference, *C*, of the spill as a function of time.

$C(r) = 2\pi r$

$C(r(t)) = 2\pi(2t)$

$= 4\pi t$

d) Express the area, *A*, of the spill as a function of time.

$A(r) = \pi r^2$

$A(r(t)) = \pi(2t)^2$

$= \pi(4t^2)$

$= 4\pi t^2$

e) Determine the change function for each of the radius, the circumference and the area. What does it tell us about the spill?

$r(t) - r(t-1) = 2r - 2(r-1)$

$= 2r - 2r + 2$

$= 2 > 0$

$\therefore$ radius is increasing (at a constant rate)

$C(t) - C(t-1) = 4\pi t - 4\pi(t-1)$

$= 4\pi t - 4\pi t + 4\pi$

$= 4\pi > 0$

$\therefore$ Circumference is increasing (at a constant rate) and > 2 $\therefore$ faster than radius

$A(t) - A(t-1) = 4\pi t^2 - 4\pi(t-1)^2$

$= 4\pi t^2 - 4\pi(t^2 - 2t + 1)$

$= 4\pi t^2 - 4\pi t^2 + 8\pi t - 4\pi$

$= 8\pi t - 4\pi$

$= 4\pi(2t - 1)$

20) Find the functions *f* and *g* such that $h(t) = f(g(x))$

a) $h(x) = (2x+1)^9$ b) $h(x) = \dfrac{1}{x^2 - 7}$ c) $h(x) = \sin(3x + \pi)$

a) $h(x) = (2x+1)^9$

$g(x) = 2x+1$

$f(x) = x^9$

b) $h(x) = \dfrac{1}{x^2-7}$

$g(x) = x^2 - 7$

$f(x) = \dfrac{1}{x}$

c) $h(x) = \sin(3x+\pi)$

$g(x) = 3x + \pi$

$f(x) = \sin x$

d) $h(x) = 4x^2 + 12x + 4$, given $f(x) = x^2 - 5$ and $g(x)$ is a linear function

$h(x) = 4x^2 + 12x + 4$

$h(x) = (2x)^2 + 6(2x) + 4$

Let $2x = a$

$h(a) = a^2 + 6a + 4 \rightarrow (a^2 + 6a + 9 - 9) + 4$

$\frac{6}{2} = 3$

$3^2 = 9$

$f(a) = a^2 - 5$

$= (a^2 + 6a + 9) - 9 + 4$

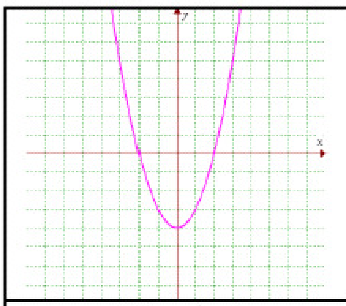
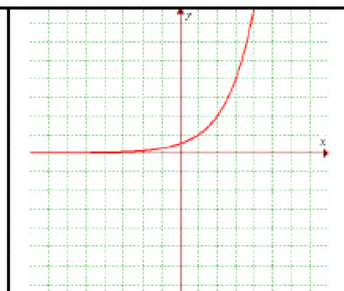
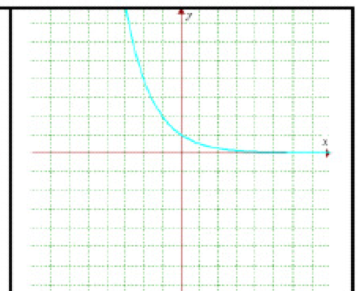
$= (a+3)^2 - 5$

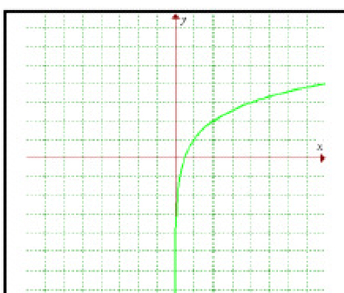
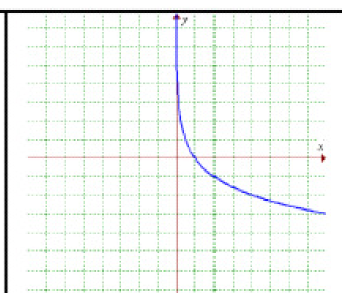
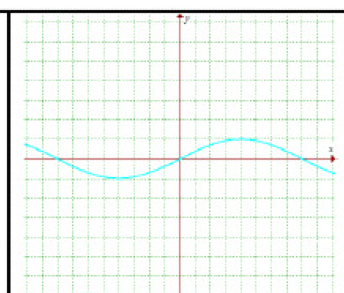
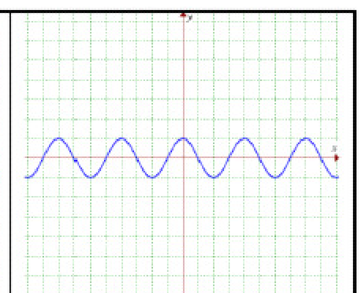
$g(a) = a + 3$

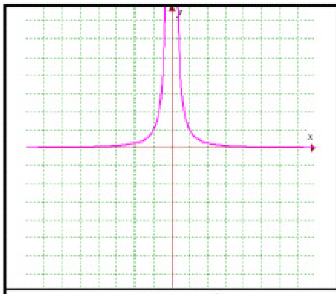
$g(x) = 2x + 3$

d) was challenging!

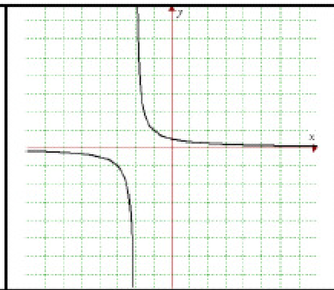
21) Complete the following table (*determine the equation of the function* **or** *draw the graph*)

			
$f(x) = x^2 - 4$	$g(x) = x^3 - 4x$	$h(x) = 2^{x-1}$	$j(x) = 2^{-x}$

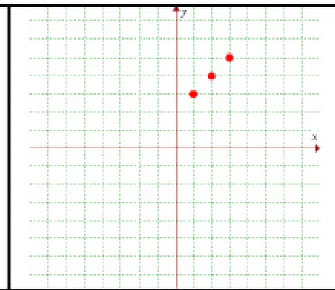
			
$k(x) = \log_2(x) + 1$	$l(x) = -\log_2 x$	$m(x) = \sin\left(\frac{1}{2}x\right)$	$n(x) = \cos(2x)$



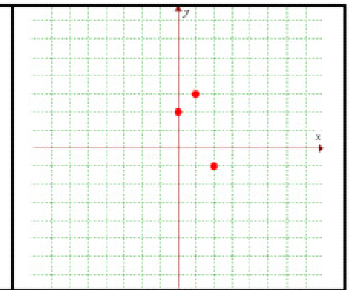
$p(x)=x^{-2}$



$q(x)=(x+2)^{-1}$



$r(x)=\{(1,\ 3),(2,\ 4),(3,\ 5)\}$



$s(x)=\{(0,\ 2),(1,\ 3),(2,\ -1)\}$

a) Determine the domain of: In general, the domain is given by the overlapping *x* values, quotient functions have extra restrictions since the denominator can not equal zero.

i) $(f+k)(x)$	ii) $(p-q)(x)$	iii) $(rs)(x)$
$D_f=\{x\mid x\in\Re\}$ $D_k=\{x\mid x>0,\ x\in\Re\}$ $D_{f+k}=\{x\mid x>0,\ x\in\Re\}$	$D_p=\{x\mid x\neq 0,\ x\in\Re\}$ $D_q=\{x\mid x\neq -2,\ x\in\Re\}$ $D_{p-q}=\{x\mid x\neq 0,-2,\ x\in\Re\}$	$D_r=\{1,2,3\}$ $D_s=\{0,1,2\}$ $D_{rs}=\{1,2\}$

v) $(m\div g)(x)$	v) $(np\div f)(x)$
$D_m=\{x\mid x\in\Re\}$ $D_g=\{x\mid x\in\Re\}$ zeros of *g*: -2, 0, 2 $D_{m\div g}=\{x\mid x\neq -2,0,2,\ x\in\Re\}$	$D_n=\{x\mid x\in\Re\}$ $D_p=\{x\mid x\neq 0,\ x\in\Re\}$ $D_q=\{x\mid x\in\Re\}$ zeros of *f*: -2, 2 $D_{p-q}=\{x\mid x\neq 0,\pm 2,\ x\in\Re\}$

b) Determine the range of:

i) $(r+s)(x)$	ii) $(f-g)(x)$
$r+s=\{(1,6),\ (2,3)\}$ $R_{r+s}=\{6,3\}$	$R_f=\{y\mid y\geq -4,\ y\in\Re\}$ $R_q=\{y\mid y\in\Re\}$ The difference of a quadratic and a cubic is a cubic $\therefore$ $R_{f-g}=\{y\mid y\in\Re\}$
iii) $(h\div j)(x)$ $R_h=\{y\mid y>0,\ y\in\Re\}$ $R_j=\{y\mid y>0,\ y\in\Re\}$ A positive divided by a positive results in a positive $R_{h\div j}=\{y\mid y>0,\ y\in\Re\}$	iv) $(fn)(x)$ $R_f=\{y\mid y\geq -4,\ y\in\Re\}$ $R_n=\{y\mid \lvert y\rvert\leq -1,\ y\in\Re\}$ numbers $\geq$ -4 will be multiplied by 1, -1 and any number in between resulting in positive and negative # $R_{fn}=\{y\mid y\in\Re\}$

c) Algebraically, determine whether the following are even, odd or neither

i) $(fp)(x)$ $(fp)(x)=\left(x^2-4\right)\left(\frac{1}{x^2}\right)$ $(fp)(-x)=\left((-x)^2-4\right)\left(\frac{1}{(-x)^2}\right)$ $(fp)(-x)=\left(x^2-4\right)\left(\frac{1}{x^2}\right)$ $(fp)(-x)=(fp)(x)$ $\therefore\ \ even$ This supports our conclusions: the product of two even functions is even.	ii) $(jn)(x)$ $(jn)(x)=2^{-x}\cdot\cos(2x)$ $(jn)(-x)=2^{-(-x)}\cdot\cos(-2x)$ $(jn)(-x)=2^{x}\cdot\cos(2x)$ $(jn)(-x)\neq(jn)(x)$ $\therefore\ \ not\ \ even$ $-(jn)(x)=-\left(2^{-x}\cdot\cos(2x)\right)$ $-(jn)(x)=-2^{x}\cdot\cos(2x)$ $-(jn)(x)\neq(jn)(-x)$ $\therefore\ \ not\ \ odd$ This supports our conclusions: the product of a "neither" with an even function is neither.
iii) $(mm)(x)$ $(mm)(x)=\sin\left(\frac{1}{2}x\right)\cdot\sin\left(\frac{1}{2}x\right)$ $(mm)(-x)=\sin\left(-\frac{1}{2}x\right)\cdot\sin\left(-\frac{1}{2}x\right)$ $(mm)(-x)=-\sin\left(\frac{1}{2}x\right)\cdot-\sin\left(\frac{1}{2}x\right)$ $(mm)(-x)=\sin\left(\frac{1}{2}x\right)\cdot\sin\left(\frac{1}{2}x\right)$ $(mm)(-x)=(mm)(x)$ $\therefore\ \ even$ This supports our conclusions: the product of two odd functions is even.	iv) $(m\div p)(x)$ $(m\div p)(x)=\sin\left(\frac{1}{2}x\right)\div\frac{1}{x^2}$ $(m\div p)(-x)=\sin\left(-\frac{1}{2}x\right)\div\frac{1}{(-x)^2}$ $(m\div p)(-x)=-\sin\left(\frac{1}{2}x\right)\div\frac{1}{x^2}$ $(m\div p)(-x)=-(m\div p)(x)$ $\therefore\ \ odd$ This supports our conclusions: the quotient of one odd function with one even function is odd.

d) Determine all the zeros for the function

i) $(fg)(x)$ zeros of f: -2, 2 zeros of g: -2, 0, 2 $\therefore$ zeros of fg: -2 (DR), 0, 2 (DR)	ii) $(m\div g)(x)$ zeros of m: 0, $\pm\pi$, $\pm 2\pi$, … in general: $\pm n\pi$ zeros of g: -2, 0, 2 $\therefore$ zeros of $m\div g$: $\pm\pi$, $\pm 2\pi$, … in general $\pm n\pi$ NOT -2 (VA), 2 (VA), 0 (hole)
iii) $(lf)(x)$ zeros of l: 1, x > 0 zeros of f: -2, 2 $\therefore$ zeros of lg: 1, 2	iv) $(h\div q)(x)$ zeros of h: none zeros of q: none $\therefore$ zeros of $m\div g$: none NOT -2 (hole)

e) Determine the average rate of change in the interval [1, 3] for the functions

i) $f(x)$ $avg \;\; RoC = \dfrac{f(3)-f(1)}{3-1}$ $= \dfrac{5-(-3)}{3-1}$ $= 4$	ii) $g(x)$ $avg \;\; RoC = \dfrac{g(3)-g(1)}{3-1}$ $= \dfrac{15-(-3)}{3-1}$ $= 9$
iii) $m(x)$ $avg \;\; RoC = \dfrac{m(3)-m(1)}{3-1}$ $\doteq \dfrac{0.9975-0.4794}{3-1}$ $\doteq 0.25905$	iv) $p(x)$ $avg \;\; RoC = \dfrac{p(3)-p(1)}{3-1}$ $= \dfrac{1/9-1}{3-1}$ $= \dfrac{-4}{9}$

f) Determine the average rate of change in the interval [1, 3] for the functions

i) $(f+g)(x)$ $avg \;\; RoC_{f+g} = avg \;\; RoC_f + avg \;\; RoC_f$ $= 4+9$ $= 13$	ii) $(gm)(x)$ This is NOT equal to the product of the two avg RoC! $(gm)(x) = (x^3-4x)\cdot(\sin\frac{1}{2}x)$ $avg \;\; RoC = \dfrac{(gm)(3)-(gm)(1)}{3-1}$ $= \dfrac{g(3)\cdot m(3)-g(1)\cdot m(1)}{3-1}$ $\doteq \dfrac{14.9625-(-1.4382)}{3-1}$ $\doteq 8.2004$
iii) $(m-p)(x)$ $avg \;\; RoC_{m-p} = avg \;\; RoC_m - avg \;\; RoC_p$ $\doteq 0.25905 - \dfrac{-4}{9}$ $\doteq 0.7035$	iv) $(f \div p)(x)$ This is NOT equal to the quotient of the two avg RoC! $(f \div p)(x) = (x^2-4)\cdot(x^{-2})$ $avg \;\; RoC = \dfrac{(f \div p)(3)-(f \div p)(1)}{3-1}$ $= \dfrac{f(3)\div p(3)-f(1)\div p(1)}{3-1}$ $= \dfrac{45-(-3)}{3-1}$ $= 24$