

## Review

Use the following functions to answer questions 1 – 5

$$f(x) = x^2 - 1, \quad g(x) = -2^x, \quad h(x) = \sin x, \quad w = \{(-2, 1), (3, 2), (4, 7)\}, \quad r(x) = \frac{x}{x+1}, \quad \text{and} \quad l(x) = 2 \log x,$$

1) Determine:

a)  $f + w$

b)  $f \cdot r$

c)  $\frac{h}{l}$

d)  $h \circ f$

e)  $g \circ w$

f)  $w \circ r$

2) Graph  $h(x)$  and  $w$ . Use your graphs to graph  $h(x) - w$ .

3) Determine:

a)  $D_{g \circ l}$

b)  $D_{l(r(x))}$

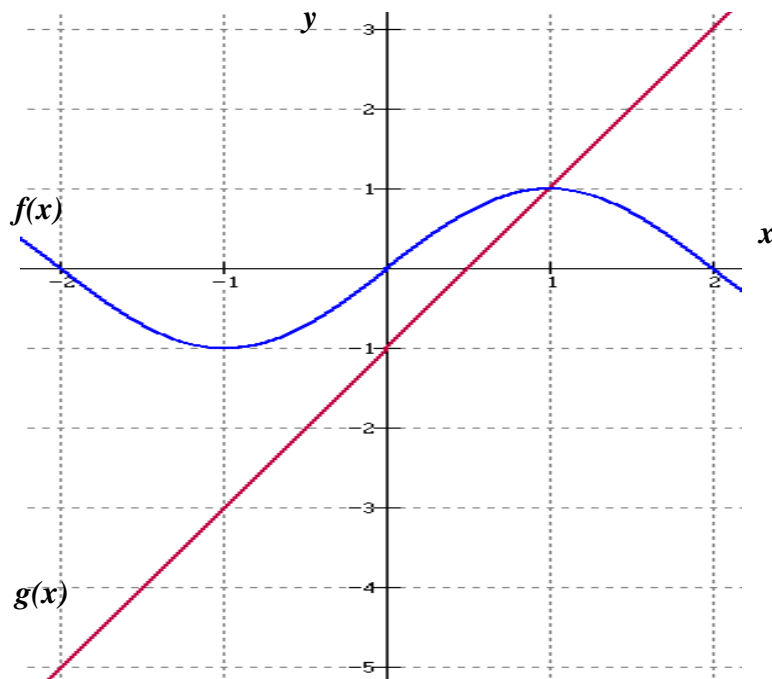
c)  $R_{h(r(x))}$

d)  $R_{w(l(x))}$

4) How many zeros does  $r(x) \cdot f(x)$  have?

5) Will the product of  $f(x)$  and  $h(x)$  be even, odd, or neither?

6) Graphs of  $y = f(x)$  and  $y = g(x)$  are given below. Sketch  $h(x) = \frac{f(x)}{g(x)}$  on the grid above.



7) Given  $f = \{(-2, 6), (-1, 8), (0, 5), (1, 0), (2, -2)\}$  and  $g(x) = x^2 + 1$ ,

a) graph  $f - g$

b) state the domain of  $f \cdot g$

c) determine  $f \circ g(x)$  when  $x = -1$

d) determine  $f(g(-1))$

e) determine the domain of  $f(g(x))$

8) If  $f(x) = \frac{x}{x-1}$  and  $g(x) = \frac{3}{x^2-1}$ , find the value of  $x$  for which  $(f + g)(x) = 1$ .

9) Given  $f(x) = x + 3$ , determine

- a)  $f^{-1}(x)$                       b)  $f \circ f^{-1}$                       c)  $f^{-1} \circ f$

10) Use composition to verify that  $f(x) = \frac{1}{x+1}$  and  $g(x) = \frac{1-x}{x}$  are inverses of each other.

11) Given  $f(x) = \sin x$  and  $g(x) = 2x - \frac{\pi}{3}$ , describe the graph of  $f \circ g$  as a transformation of the graph of  $f$ .

12) Given  $q(x) = \frac{2x-1}{x+5}$ ,

- a) determine the intercept(s), asymptote(s), intervals of increase/decrease, and end behaviour of the function,  
b) sketch the graph of the function,  
c) define an equation  $f(x)$  and an equation  $g(x)$  such that  $q(x) = \frac{f(x)}{g(x)}$   
d) justify the properties you found in a) by studying the properties of the functions  $f(x)$  and  $g(x)$ .

13) Given  $f(x) = \sec x$ , where  $-2\pi \leq x \leq 2\pi$  and  $g(x) = \log x$  determine,

- a) the domain of  $\frac{f}{g}$   
b) at most, the number of zeros for  $\frac{f}{g}$ , do you think this number is accurate?  
c) the domain of  $\frac{g}{f}$   
d) at most, the number of zeros for  $\frac{g}{f}$ , do you think this number is accurate?

14) Given  $f(x) = 3^x$  and  $g(x) = \tan x$ , where  $-2\pi \leq x \leq 2\pi$  determine,

- b) the domain of  $f \cdot g$   
c) the range of  $f \cdot g$   
d) at most, the number of zeros for  $f \cdot g$ , do you think this number is accurate?

15) The plucked string of a guitar and the sound as it fades away can be represented by a damped sine wave that has an equation of the form  $y = 2^{-t} \sin \pi t$ . Sketch a graph of the functions for  $0 \leq t \leq 2\pi$ .

16) Let  $S(t)$  represent the number of single adults in Canada in year  $t$  and  $M(t)$  represent the number of married adults in Canada in year  $t$ . Let  $E(t)$  represent the average amount spent on entertainment by a single adult and let  $N(t)$  represent the average amount spent on entertainment by a married adult. Using a combination of the functions defined above come up with representations for the following functions:

- a)  $A(t)$ , the number of Canadian adults in Canada in year  $t$ .  
b)  $B(t)$ , the amount of money spent on entertainment by Canadian single adults in year  $t$ .  
c)  $C(t)$ , the amount of money spent on entertainment by Canadian adults in year  $t$ .

17) The *change function* is defined as  $d(x) = f(x) - f(x-1)$ . What is the meaning, in terms of  $f$ , if

- a)  $d(x) > 0$                       b)  $d(x) < 0$                       c)  $d(x) = 0$

18) Each of the following graphs is a combination of two of the functions:  $f(x) = x$ ,  $g(x) = 2^x$ ,  $h(x) = x^2$ ,  $j(x) = \cos x$  and one of the operations: addition, subtraction, multiplication, division, for each of the given graphs. State the equation and explain how you know using key features of the functions.

Graph	Equation and Explanation

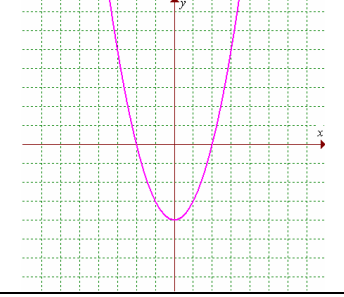
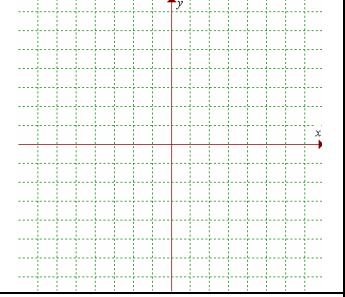
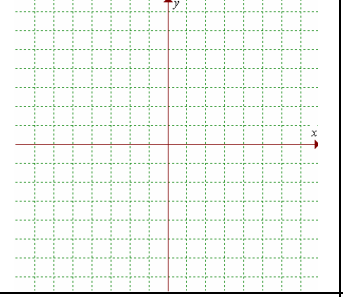
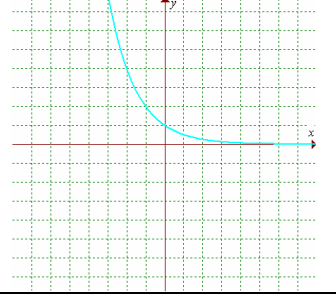
19) Suppose that some oil has been spilled in water and has formed a circular oil slick. One minute after the spill the radius of the slick is 2 metres and 3 minutes after the spill the radius is 6 metres.

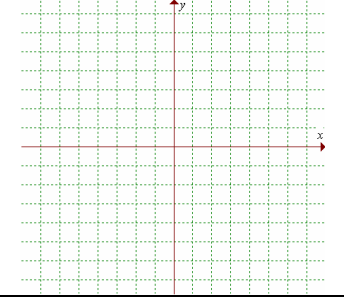
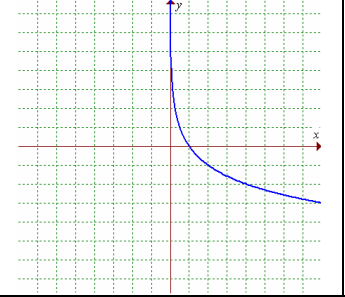
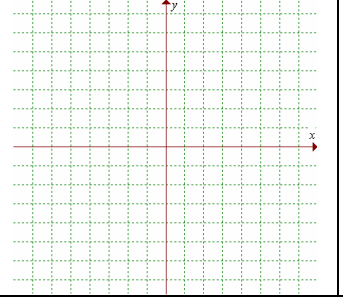
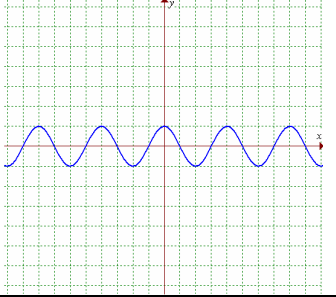
- Express the radius,  $r$ , of the spill as a function of time,  $t$ , if the radius is increasing at a constant rate.
- Was the radius 0 at time  $t = 0$ ?
- Express the circumference,  $C$ , of the spill as a function of time.
- Express the area,  $A$ , of the spill as a function of time.
- Determine the change function for each of the radius, the circumference and the area. What does it tell us about the spill?

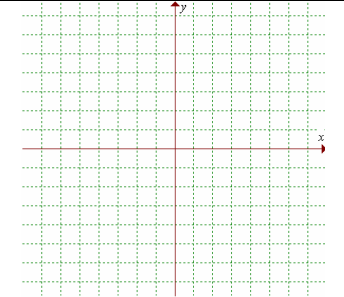
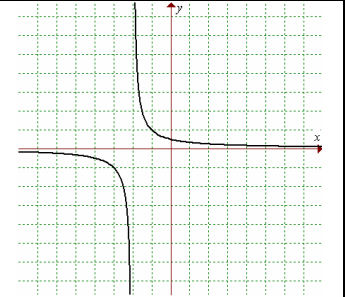
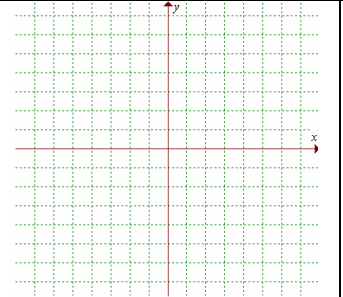
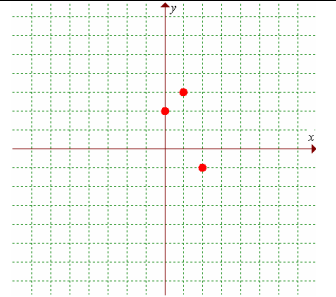
20) Find the functions  $f$  and  $g$  such that  $h(t) = f(g(x))$

- $h(x) = (2x + 1)^9$
- $h(x) = \frac{1}{x^2 - 7}$
- $h(x) = \sin(3x + \pi)$
- $h(x) = 4x^2 + 12x + 4$ , given  $f(x) = x^2 - 5$  and  $g(x)$  is a linear function

21) Complete the following table (determine the equation of the function or draw the graph)

			
$f(x) =$	$g(x) = x^3 - 4x$	$h(x) = 2^{x-1}$	$j(x) =$

			
$k(x) = \log_2 x + 1$	$l(x) =$	$m(x) = \sin\left(\frac{1}{2}x\right)$	$n(x) =$

			
$p(x) = x^{-2}$	$q(x) =$	$r(x) = \{(1, 3), (2, 4), (3, 5)\}$	$s(x) =$

a) Determine the domain of:

- i)  $(f+k)(x)$    ii)  $(p-q)(x)$    iii)  $(rs)(x)$    iv)  $(m \div g)(x)$    v)  $(np \div f)(x)$

b) Determine the range of:

- i)  $(r+s)(x)$    ii)  $(f-g)(x)$    iii)  $(h \div j)(x)$    iv)  $(fn)(x)$

c) Algebraically, determine whether the following are even, odd or neither

- i)  $(fp)(x)$    ii)  $(jn)(x)$    iii)  $(mm)(x)$    iv)  $(m \div p)(x)$

d) Determine all the zeros for the function

- i)  $(fg)(x)$    ii)  $(m \div g)(x)$    iii)  $(lf)(x)$    iv)  $(h \div q)(x)$

e) Determine the average rate of change in the interval  $[1, 3]$  for the functions

- i)  $f(x)$    ii)  $g(x)$    iii)  $m(x)$    iv)  $p(x)$

f) Determine the average rate of change in the interval  $[1, 3]$  for the functions

- i)  $(f+g)(x)$    ii)  $(gm)(x)$    iii)  $(m-p)(x)$    iv)  $(f \div p)(x)$