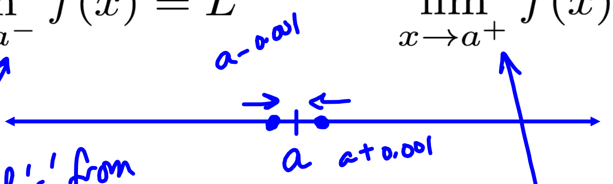
Defining the Limit

$$\lim_{x \to a} f(x) = L$$

"the limit of $f(x)$ as x approaches a equals L"

Consider a possible meaning for:

$$\lim_{x \to a^-} f(x) = L \qquad \lim_{x \to a^+} f(x) = L$$

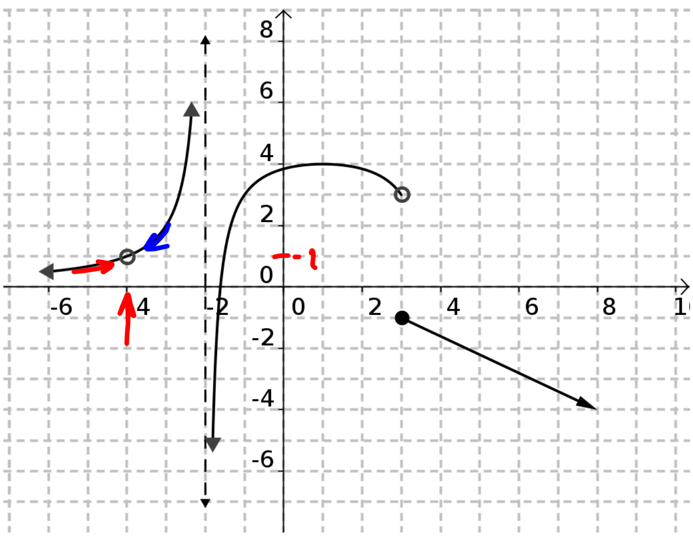


approach 'a' from
→ left side
→ below (lower value)

approach from
→ right
→ above (higher value)

Feb 5-10:21 PM

Ex.1 Evaluate or explain each limit for the graph (see worksheet)



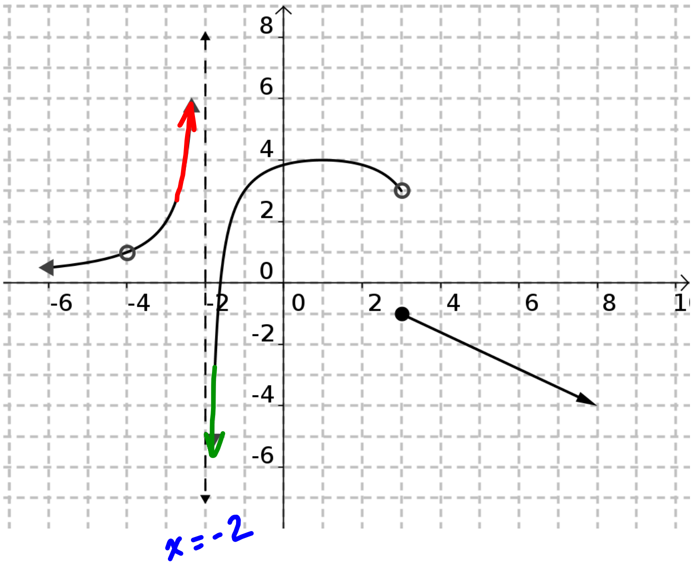
$\lim_{x \to -4^-} f(x) = -1$

$\lim_{x \to -4^+} f(x) = -1$

$\lim_{x \to -4} f(x) = -1$

Feb 5-10:54 PM

Ex.1 Evaluate or explain each limit for the graph
(see worksheet)



"is"

$$\lim_{x\to -2^-} f(x) = +\infty$$

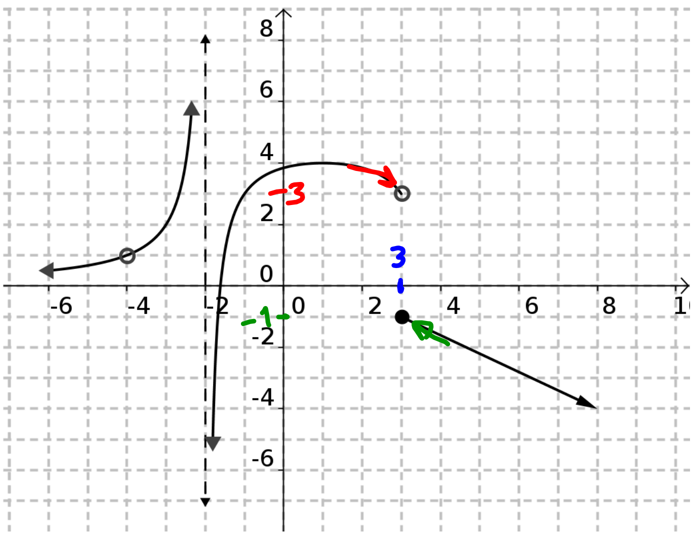
$$\lim_{x\to -2^+} f(x) = -\infty$$

$$\lim_{x\to -2} f(x) \quad \text{DNE}$$

(does not exist)

Feb 5-10:54 PM

Ex.1 Evaluate or explain each limit for the graph
(see worksheet)



$$\lim_{x\to 3^-} f(x) = 3$$

$$\lim_{x\to 3^+} f(x) = -1$$

$$\lim_{x\to 3} f(x) \quad \text{DNE}$$

Feb 5-10:54 PM

Summary:

(1) $\lim\limits_{x \to a} f(x)$ may exist even if $f(a)$ is undefined

e.g., $x = a$ is a hole or VA

(2) The limit exists if and only if the limiting value from the left is equal to the limiting value from the right.

(3) The limit may be positive or negative infinity, but only if each of the one-sided limits gives the same result.

For example:

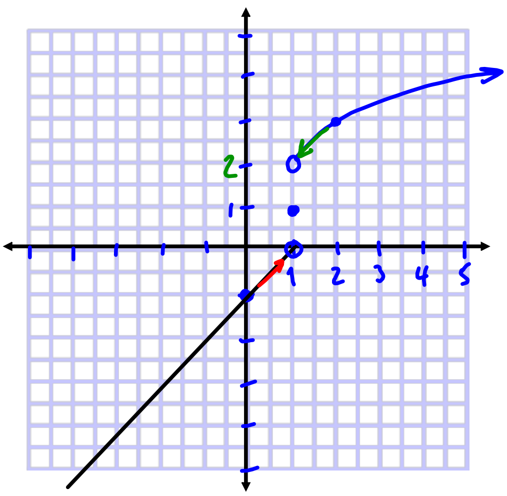
If $\lim\limits_{x \to a^-} f(x) = +\infty$ and $\lim\limits_{x \to a^+} f(x) = +\infty$

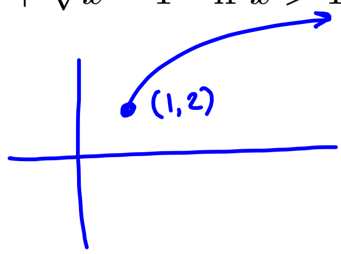
then $\lim\limits_{x \to a} f(x) = +\infty$

Feb 6-12:18 AM

Ex.2 Sketch a graph of the piecewise function and then evaluate the limit.

$\lim\limits_{x \to 1} f(x)$ for $f(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ 2 + \sqrt{x-1} & \text{if } x > 1 \end{cases}$





$\lim\limits_{x \to 1^-} f(x) = 0$

$\lim\limits_{x \to 1^+} f(x) = 2$

$\lim\limits_{x \to 1} f(x)$ DNE

Feb 6-12:03 AM

Assigned Work:

p.38 # 5, 6, 7, 10def, 11bc, 12a

Feb 6-8:30 AM