Properties of Limits

For any real number a, if f and g both have limits that exist at $x = a$, the limit may be evaluated using:

1. $\lim_{x\to a} k = k$, for any real constant k
2. $\lim_{x\to a} x = a$
3. $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
4. $\lim_{x\to a}[cf(x)] = c\left[\lim_{x\to a} f(x)\right]$ → number
5. $\lim_{x\to a}[f(x)g(x)] = [\lim_{x\to a} f(x)][\lim_{x\to a} g(x)]$
6. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
7. $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$

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Properties of Limits

Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 2} \dfrac{x-2}{4-x^2}$

(b) $\lim_{x\to 3} \dfrac{|x-3|}{x-3}$

(c) $\lim_{x\to 0} \dfrac{\sqrt{x+3}-\sqrt{3-x}}{x}$

(d) $\lim_{x\to 0} \dfrac{(x+8)^{\frac{1}{3}}-2}{x}$

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Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(a) $$\lim_{x\to 2}\frac{x-2}{4-x^2}$$

$\frac{0}{0}$

$x-2$
$= -2+x$
$= -1(2-x)$

$$= \lim_{x\to 2}\frac{\cancel{(x-2)}}{\cancel{(2-x)}(2+x)}$$

$$= \lim_{x\to 2}\frac{-1\cancel{(2-x)}}{\cancel{(2-x)}(2+x)}$$

$$= \lim_{x\to 2}\frac{-1}{2+x}$$

$$= \frac{-1}{4}$$

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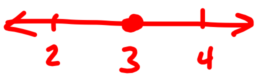
Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(b) $$\lim_{x\to 3}\frac{|x-3|}{x-3}$$

$$|x| = \begin{cases} -x, & x<0 \\ x, & x\geq 0 \end{cases}$$

$|-3| = -(-3)$
$= 3$



$$\lim_{x\to 3^-}\frac{-\cancel{(x-3)}}{\cancel{(x-3)}}$$

$= -1$

$$\lim_{x\to 3^+}\frac{\cancel{(x-3)}}{\cancel{(x-3)}}$$

$= 1$

$\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$ $\therefore$ limit DNE

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Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(c) $\lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3-x}}{x} \times \frac{\sqrt{x+3}+\sqrt{3-x}}{\sqrt{x+3}+\sqrt{3-x}}$

$\frac{\sqrt{3}-\sqrt{3}}{0} = \frac{0}{0}$

$= \lim_{x\to 0}\frac{(\sqrt{x+3})^2-(\sqrt{3-x})^2}{x(\sqrt{x+3}+\sqrt{3-x})}$

$= \lim_{x\to 0}\frac{(x+3)-(3-x)}{x(\sqrt{x+3}+\sqrt{3-x})}$

$= \lim_{x\to 0}\frac{2\cancel{x}^{1}}{{}_{1}\cancel{x}(\sqrt{x+3}+\sqrt{3-x})}$

$= \lim_{x\to 0}\frac{2}{\sqrt{x+3}+\sqrt{3-x}}$

$= \frac{2}{\sqrt{3}+\sqrt{3}}$

$= \frac{\cancel{2}^{1}}{{}_{1}\cancel{2}\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

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Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(d) $\lim_{x\to 0}\frac{(x+8)^{\frac{1}{3}}-2}{x}$

$\frac{0}{0}$

let $u=(x+8)^{\frac{1}{3}}$

$u^3 = x+8$

$u^3-8 = x$

as $x\to 0$, $\underbrace{u \to (0+8)^{\frac{1}{3}}=2}_{u\to 2}$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$= \lim_{u\to 2}\frac{u-2}{u^3-8}$

$= \lim_{u\to 2}\frac{\cancel{(u-2)}^{1}}{\cancel{(u-2)}_{1}(u^2+2u+4)}$

$= \lim_{u\to 2}\frac{1}{u^2+2u+4}$

$= \frac{1}{4+4+4}$

$= \frac{1}{12}$

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In Summary:

To help evaluate limits we can:

- substitute directly (not shown)
- factor (as in a)
- rationalize (as in c)
- use one-sided limits (as in b)
- use a change of variable (as in d) *

Assigned Work:

p.45 # 4, 7bce, 8cef, 9, 10ad

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