Derivatives - The Power Rule

Recall, at $x = a$:

$$m_{\text{tangent}} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

Now, the derivative of $f(x)$ at $x = a$:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

or

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

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The Power Rule:

Given $f(x) = x^n$, where $n \in \mathbb{R}$

then $f'(x) = nx^{n-1}$

where $f'(x)$ is (a) the slope of the tangent at x
or (b) the instantaneous RoC at x

Ex.1 Find the derivative of each function.

Hint: Express all terms as powers of x

(a) $y = 3x^2 - 5x + 7$

(b) $y = x(x-4)(x+4)$

(c) $y = \dfrac{5}{x^2}$

(d) $y = \sqrt{2x^3}$

Jan 23-8:35 PM

Notation used for the derivative of a function:

y' " y prime "

$y = x^3, \quad y' = 3x^2$

$\dfrac{dy}{dx}$ the differential of y with respect to the differential of x

$\dfrac{dy}{dx} = 3x^2$

$f'(x)$ " f prime of x "

$f(x) = x^3$

$\dfrac{d}{dx} f(x)$ Leibniz notation: the derivative of $f(x)$ with respect to x

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Constant Rule: If $f(x) = k$ then $f'(x) = 0$

$$\boxed{\text{Power Rule: If } f(x) = x^n \text{ then } f'(x) = nx^{n-1}}$$

Constant Multiple Rule: If $f(x) = kg(x)$ then $f'(x) = kg'(x)$

Sum Rule: If $f(x) = g(x) + h(x)$, then $f'(x) = g'(x) + h'(x)$

Difference Rule: If $f(x) = g(x) - h(x)$, then $f'(x) = g'(x) - h'(x)$

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Constant Rule: $\frac{d}{dx}[k]=0$

Power Rule: $\boxed{\frac{d}{dx}\left[x^n\right]=nx^{n-1}}$

Constant Multiple Rule: $\frac{d}{dx}[kf(x)]=k\frac{d}{dx}[f(x)]$

Sum Rule: $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$

Difference Rule: $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$

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Ex.1 Find the derivative of each function.

Hint: Express all terms as powers of x

(a) $y=\underbrace{3x^2}_{f(x)}-\underbrace{5x}_{g(x)}+\underbrace{7}_{h(x)}$

$y'=6x-5$

$f'(x)=6x^1$
$=6x$

$g'(x)=5x^0$
$=5$

$h'(x)=0$

(b) $y=x(x-4)(x+4)$

$y=x(x^2-16)$

$y=x^3-16x$

$\frac{dy}{dx}=3x^2-16x^0$
$=3x^2-16$

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Ex.1 Find the derivative of each function. $f(x) = x^n$

Hint: Express all terms as powers of x

(c) $y = \dfrac{5}{x^2}$

$= 5x^{-2}$

$y' = -10x^{-3}$ ✓

$= \dfrac{-10}{x^3}$ ✓

(d) $y = \sqrt[2]{2x^3}$

$y = (2x^3)^{\frac{1}{2}}$

$= 2^{\frac{1}{2}}(x^3)^{\frac{1}{2}}$

$= \sqrt{2}\, x^{\frac{3}{2}}$

$\dfrac{dy}{dx} = \sqrt{2}\left(\dfrac{3}{2}x^{\frac{1}{2}}\right)$

$= \dfrac{3\sqrt{2}}{2}x^{\frac{1}{2}}$ ✓

$= \dfrac{3\sqrt{2}}{2}\sqrt{x}$

$= \dfrac{3\sqrt{2x}}{2}$ ✓

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Ex.2 Find the point on the curve $y = x^2 + 3x - 10$ where the slope of the tangent is equal to -1. Sketch this situation.

$= (x+5)(x-2)$

$m_{tan} = -1$

$f'(x) = -1$

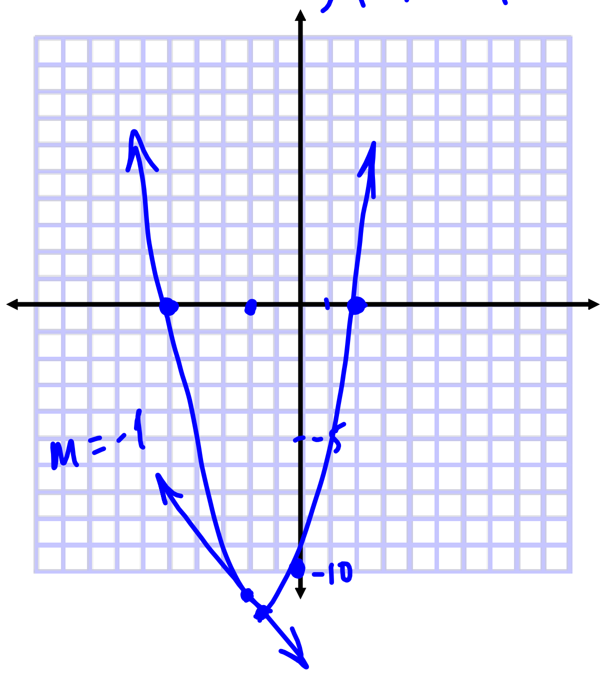
$y' = 2x + 3$

set $y' = -1$

$-1 = 2x + 3$

$-4 = 2x$

$x = -2$



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Assigned Work:

summarize table on p.81 into your notes
read proofs on p.76, p.77
p.82 # 2, 3def, 4cef, 8ab, 9bf, 14, 15, 16

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9(f) $y = \frac{\sqrt{x} - 2}{\sqrt[3]{x}} = \frac{\sqrt{x}}{\sqrt[3]{x}} - \frac{2}{\sqrt[3]{x}}$

equation of tangent line @ P(1,-1)

$y = mx + b$
OR
$Ax + By + C = 0$

$m_{tan} = iRoC$
$= y'$ at $x = 1$

$y = \frac{\sqrt{x} - 2}{\sqrt[3]{x}}$
$= \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} - \frac{2}{x^{\frac{1}{3}}}$
$= x^{\frac{1}{6}} - 2x^{-\frac{1}{3}}$

$\frac{1}{2} - \frac{1}{3}$
$= \frac{3}{6} - \frac{2}{6}$
$= \frac{1}{6}$

$y' = \frac{1}{6}x^{-\frac{5}{6}} - 2\left(-\frac{1}{3}x^{-\frac{4}{3}}\right)$
$y' = \frac{1}{6}x^{-\frac{5}{6}} + \frac{2}{3}x^{-\frac{4}{3}}$

m_{tan} at $x = 1$
$y' = \frac{1}{6}(1)^{-\frac{5}{6}} + \frac{2}{3}(1)^{-\frac{4}{3}}$
$= \frac{1}{6} + \frac{2}{3}$
$= \frac{1}{6} + \frac{4}{6}$
$m_{tan} = \frac{5}{6}$

$y = \frac{5}{6}x + b$
sub P(1,-1)
$b =$ ____

Feb 10-10:38 AM