The Product Rule

If $h(x) = f(x)g(x)$, then

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

Proof from first principles (i.e., limit definition) on p.86.

Similarly, if $y = f(x)g(x)h(x)$, then

$$y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

Feb 13-7:57 AM

Ex.1 Find the derivative of each function (using the product rule).

(a) $y = (2x+4)(3x-5)$

(b) $f(x) = (3x^2 + 4x - 6)(2x^2 - 3x - 9)$

$h'(x) = f'(x)g(x) + f(x)g'(x)$

(a) $y = \underbrace{(2x+4)}_{f(x)}\underbrace{(3x-5)}_{g(x)}$

$y' = f'(x)g(x) + f(x)g'(x)$

$y' = (2)(3x-5) + (2x+4)(3)$

$= 6x - 10 + 6x + 12$

$= 12x + 2$

Feb 13-8:08 AM

Ex.1 Find the derivative of each function (using the product rule).

(a) $y=(2x+4)(3x-5)$

(b) $f(x)=(3x^2+4x-6)(2x^2-3x-9)$

$\qquad\qquad g(x) \qquad\qquad h(x)$

$$f'(x) = g'(x)\,h(x) + g(x)\,h'(x)$$

$$f'(x) = (6x+4)(2x^2-3x-9)+(3x^2+4x-6)(4x-3)$$

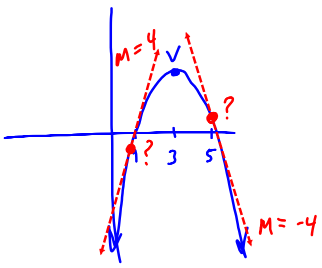
$$= 12x^3-18x^2-54x+8x^2-12x-36$$
$$+12x^3+16x^2-24x-9x^2-12x+18$$

$$= 24x^3-3x^2-102x-18$$

Feb 13-8:08 AM

Ex.2 Find the point(s) on the curve which satisfy:

$f(x)=2(x-1)(5-x)$ and $|m_{\text{tangent}}|=4$



$m_T = 4$ or $m_T = -4$

$m_{tangent} = f'(x)$

$f'(x) = 2[(1)(5-x)+(x-1)(-1)]$

$= 2[5-x-x+1]$

$= 2[-2x+6]$

$f'(x) = -4x+12$

solve: $m_T = 4$

$4 = -4x+12$	$-4 = -4x+12$
$-8 = -4x$	$-16 = -4x$
$x = 2$	$x = 4$

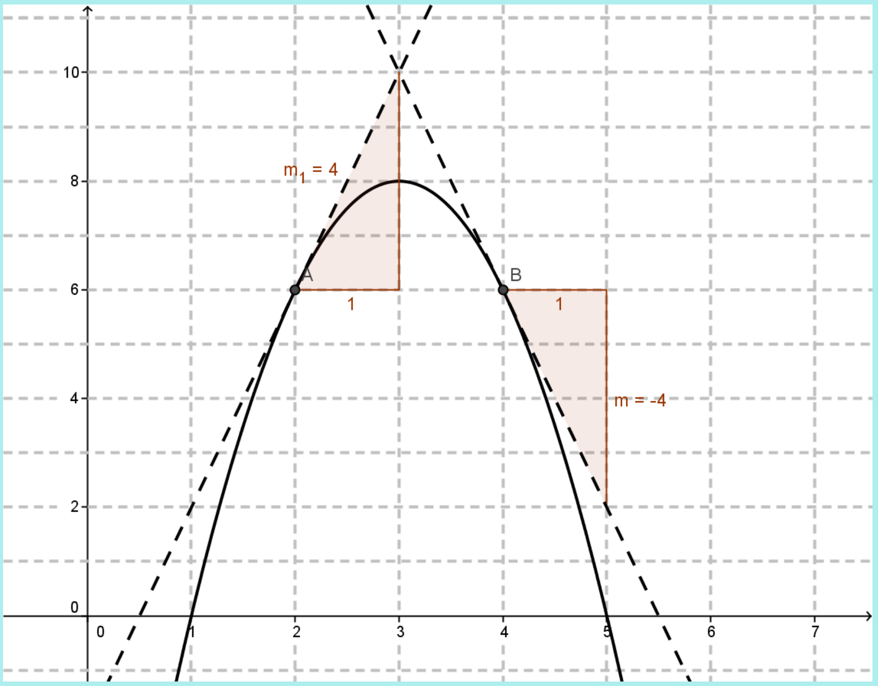
$f(2) = 2(2-1)(5-2)$	$f(4) = 2(4-1)(5-4)$
$= 2(1)(3)$	$= 2(3)(1)$
$= 6$	$= 6$

$\therefore |m_T| = 4$ at $(2,6)$ and $(4,6)$

Feb 13-8:19 AM

Ex.2 Find the point(s) on the curve which satisfy:

$$f(x) = 2(x-1)(5-x) \text{ and } \left| m_{\text{tangent}} \right| = 4$$



Feb 13-8:19 AM

Assigned Work:

p. 90 #1abcde, 5abc, 6, 7, 9, 12

7. horizontal tangent $\rightarrow m_{tan} = 0$

$\rightarrow f'(x) = 0$

① find $f'(x)$

② solve $f'(x) = 0$ for x

Feb 13-8:09 AM

9. $V(t) = 75\left(1-\frac{t}{24}\right)^2 \quad 0 \le t \le 24$

how quickly (iRoC) when 60% full

① when? set $V(t) = 60\%$ of $75L$
$= 45$

solve for t

$$45 = 75\left(1-\frac{t}{24}\right)^2$$
$$\frac{45}{75} = \left(1-\frac{t}{24}\right)^2$$
$$\pm\sqrt{\frac{45}{75}} = 1-\frac{t}{24}$$
$$\vdots$$
$$t = \underset{\checkmark}{\underline{t_1}} \text{ or } \underset{\times}{\underline{t_2}} \quad \text{in domain?}$$

② iRoC at t_1, $V'(t)$
$\rightarrow V'(t_1) =$ ______

Feb 13-9:24 AM

12. $f(x) = ax^2 + bx + c$

$P_1(2,19) \qquad P_2 = (-1,-8)$

① Sub P_1 and P_2 $\rightarrow$ 2 equations in a, b, c

② $f'(x) = m_{tan}$

$m = 0$ at P_2

$f'(x_2) = 0$

③ solve system: 3 eq, 3 unknowns.

Feb 13-9:30 AM