Linear Combinations & Spanning Sets

Apr. 30/2014

Given noncollinear vectors $\vec{u}$ and $\vec{v}$, a linear combination of these vectors is:

$$a\vec{u} + b\vec{v}$$

where a and b are scalars.

Ex.1 Show that the vector, $\vec{w} = (4, 23)$ can be written as a linear combination of $\vec{u} = (-1, 4)$ and $\vec{v} = (2, 5)$

$$\vec{w} = a\vec{u} + b\vec{v}$$
$$(4,23) = a(-1,4) + b(2,5)$$
$$4\vec{i} + 23\vec{j} = -a\vec{i} + 4a\vec{j} + 2b\vec{i} + 5b\vec{j}$$

x-dir

$4 = -a + 2b$ ①

y-dir

$23 = 4a + 5b$ ②

$4\times$①: $16 = -4a + 8b$

$39 = 13b$

$\boxed{b = 3}$

Sub $b = 3$ into ①

$4 = -a + 2(3)$

$\boxed{a = 2}$

$\therefore \vec{w} = 2\vec{u} + 3\vec{v}$

LS = RS to check.

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Ex.1 Show that the vector, $\vec{w} = (4, 23)$ can be written as a linear combination of $\vec{u} = (-1, 4)$ and $\vec{v} = (2, 5)$

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Every vector in R^2 can be written uniquely as a linear combination of the unit vectors, $\vec{i}$ and $\vec{j}$.

This is actually true for any pair of nonzero, noncollinear vectors in the x-y plane. Such a pair of vectors is called a spanning set in R^2.

Ex.2 Prove that the vectors {(2,1), (-3,-1)} form a spanning set in R^2.

→ any vector in R^2, (x,y),
can be written as a linear combination
$(x,y) = a(2,1) + b(-3,-1)$

x-dir $\quad$ y-dir

$x = 2a - 3b$ ① $\quad$ $y = a - b$ ②

$2y = 2a - 2b$ ② × 2

$x - 2y = -b$

$b = -x + 2y$

$y = a - (-x + 2y)$

$y = a + x - 2y$

$a = -x + 3y$

∵ x & y are real values.

∴ a & b will also be real for any (x,y)

∴ any (x,y) can be formed by $a\vec{u} + b\vec{v}$.

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Ex.2 Prove that the vectors {(2,1), (-3,-1)} form a spanning set in R^2.

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In R^3, any pair of nonzero, noncollinear vectors will span a plane (but not necessarily the x-y plane). Any vector in the same plane can be expressed as a linear combination of those vectors.

Corollary: Any vector which is a linear combination of those vectors must lie in the same plane.

Any vector in R^3 can be written uniquely as a linear combination of the standard basis vectors.

$$\overrightarrow{OP} = (a,b,c) = a(1,0,0) + b(0,1,0) + c(0,0,1)$$

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Ex.3 Show that the vector (-9,-4,1) lies in a plane defined by the vectors (-1,-2,1) and (3,-1,1).

$(-9,-4,1) = a(-1,-2,1) + b(3,-1,1)$

$-9\vec{i} - 4\vec{j} + \vec{k} = -a\vec{i} - 2a\vec{j} + a\vec{k} + 3b\vec{i} - b\vec{j} + b\vec{k}$

x: $-9 = -a + 3b$ ①
$1 = a + b$
$-8 = 4b$
$\boxed{b = -2}$

y: $-4 = -2a - b$ ②

z: $1 = a + b$ ③
$1 = a + (-2)$
$\boxed{3 = a}$

∵ these values must work for x, y, and z directions,

option 1: test ② (not used at all)

✓ option 2: test LS/RS for full expression

$LS = (-9,-4,1)$

$RS = a(-1,-2,1) + b(3,-1,1)$
$= 3(-1,-2,1) + (-2)(3,-1,1)$
$= (-3,-6,3) + (-6,2,-2)$
$= (-9,-4,1)$

$LS = RS$ ✓

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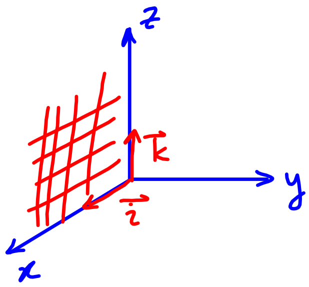
Ex.4 The vector set $\{\overset{\vec{i}}{(1, 0, 0)}, \overset{\vec{k}}{(0, 0, 1)}\}$ spans a plane in $\mathbf{R}^3$. What is the $\underbrace{\text{equation of the plane}}_{x\text{-}z \text{ plane}}$?



any point on x-z plane

$P(x, 0, z)$

$\overrightarrow{OP} = (x, 0, z)$

eqn of x-z plane is $y = 0$

in R^2, $y=0$ is a line

in R^3, $y=0$ is the x-z plane

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Assigned Work:

p.340 # 6, 7b, 8, 10, 11, $\underset{a}{13}$, 14

Review:

p.344 - 347 # 1, 2a, 3, 5, 6a, 7, 8,
11, 12b, 15ab,
16c(also find direction),
19a, 21, 23

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6. $\underbrace{\{\underbrace{(-1,2),(2,-4)},(-1,1),(-3,6),(1,0)\}}_{\text{any vector } (x,y) \text{ in } R^2}$

$(2,-4) = -2(-1,2)$

collinear

A	B	C
$(-1,2)$	$(-1,1)$	$(1,0)$
$(2,-4)$		
$(-3,6)$		

$\{A,B\}$ $\{A,C\}$ $\{B,C\}$

valid answers:

$\{(-1,2),(-1,1)\}$

$\{(2,-4),(-1,1)\}$

$\{(-1,1),(1,0)\}$

$\vdots$

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8. $\underbrace{\text{span } x\text{-}y}_{\text{z-component is 0.}}$ in $\underbrace{R^3}_{(x,y,z)}$

$(x,y,0)$

infinite # solutions

$\{(1,1,0),(2,3,0)\}$

show $(-1,2,0) = a(1,1,0) + b(2,3,0)$

$\underline{x}$

$-1 = a + 2b$

$-1 = a + 2(3)$

$a = -7$

$\underline{y}$

$2 = a + 3b$

$\underline{-1 = a + 2b}$

$3 = b$

$\therefore (-1,2,0) = -7(1,1,0) + 3(2,3,0)$

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13(a)

try $(-1,2,3) = a(4,1,-2) + b(-14,-1,16)$

$-1 = 4a - 14b \qquad 2 = a - b \qquad 3 = -2a + 16b$

$\times 2 \qquad 4 = 2a - 2b$

$+ \quad 7 = 14b$

$2 = a - \frac{1}{2} \longleftarrow b = \frac{1}{2}$

$a = \frac{5}{2}$

$LS = -1 \qquad RS = 4\left(\frac{5}{2}\right) - 14\left(\frac{1}{2}\right)$

$= 10 - 7$

$= 3$

$LS \neq RS$

$\therefore$ solution does NOT apply to this linear combination

$\therefore$ Vectors are NOT in the same plane.

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14. $(-5,6,x) = a(-1,3,4) + b(-2,3,-1)$

$-5 = -a - 2b \qquad 6 = 3a + 3b \qquad x = 4a - b$

$2 = a + b \qquad 2 = a + b$

$-3 = -b \qquad 2 = a + 3$

$\boxed{b = 3} \qquad \boxed{a = -1}$

$x = 4(-1) - (3)$

$x = -4 - 3$

$\boxed{x = -7}$

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