Velocity Vectors

Velocity is a vector, having both magnitude and direction.

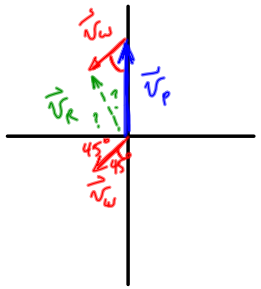
There are scenarios where two or more velocity vectors can combine to form a resultant vector. For example, a boat in a current, or a plane on a windy day.

To solve such problems, remember to:

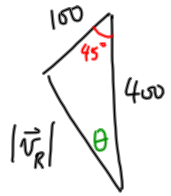
(1) draw a neat diagram showing the vectors and how they relate to each other.

(2) make sure you know if you are interested in the resultant vector, or one of the component vectors.

Ex. A plane heads north with an air speed of 400 km/h. It is blown off course by a wind of 100 km/h from the northeast. Determine the resultant ground velocity of the plane.



$\vec{v}_R = \vec{v}_{ground}$



$|\vec{v}_R|^2 = 100^2 + 400^2 - 2(100)(400)\cos 45°$

$|\vec{v}_R|^2 = 170000 - 80000\left(\frac{1}{\sqrt{2}}\right)$

$|\vec{v}_R| \doteq 336.8$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

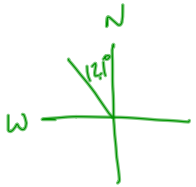
$\frac{\sin\theta}{100} = \frac{\sin 45°}{|\vec{v}_R|}$

$\sin\theta = \frac{100\left(\frac{1}{\sqrt{2}}\right)}{|\vec{v}_R|}$

$\theta \doteq 12.1°$ or $\theta \doteq 180° - 12.1°$

$\doteq 167.9°$ (reject)

but $167.9° + 45° > 180°$ not possible in Δ



∴ ground velocity is 336.8 km/h [N 12.1° W]

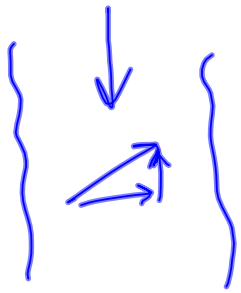
Terminology:

Air speed: The speed of the object in still air. There is no equivalent term for water, but we say phrases like, "speed in still water" to mean the same.

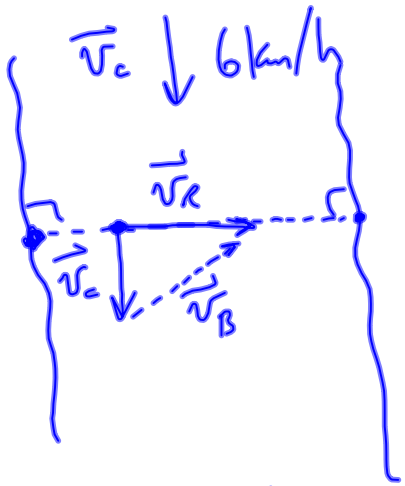
Ground speed: The speed of the object after taking into account other effects (e.g., wind, current).

N/S/E/W Wind: A "west wind" or "westerly wind" is blowing towards the east . Could also be said as "a wind from the west."

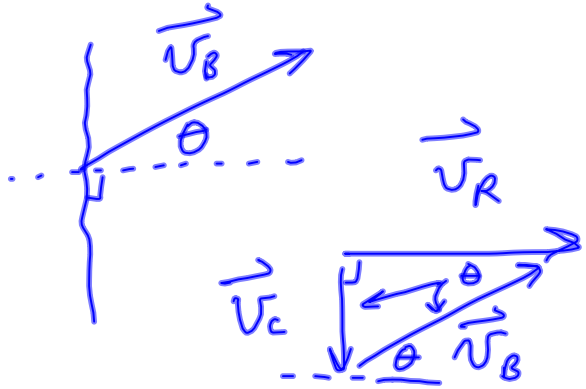
Upstream/Downstream: "Upstream" means to go against the current. "Upwind" has a similar meaning in the air. Does not necessarily mean directly opposite.



Ex. A river is 2 km wide and flows at 6 km/h. A motorboat, which can travel at 20 km/h in still water, wants to each the opposite shore directly across the river. How should the boat be steered?



$|\vec{v}_B| = 20$ angle?



$$\sin\theta = \frac{|\vec{v}_c|}{|\vec{v}_B|}$$

$$\sin\theta = \frac{6}{20}$$

$$\theta \doteq 17.5^\circ$$

$\therefore$ steer the boat 17.5° upstream.

Assigned Work:

p.369 # 3, 4, 6, 7, 9, 10, 11, 14