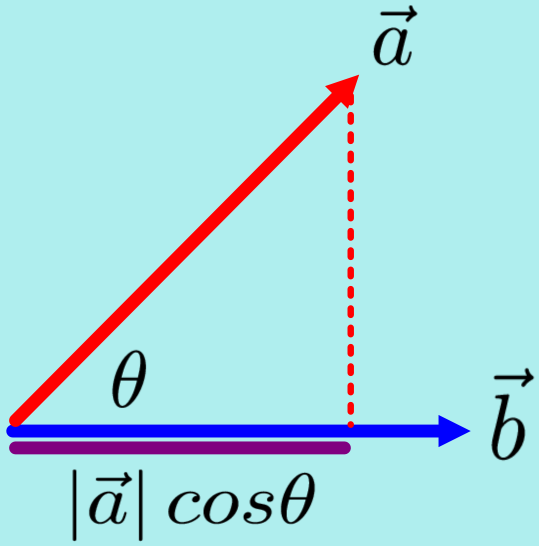
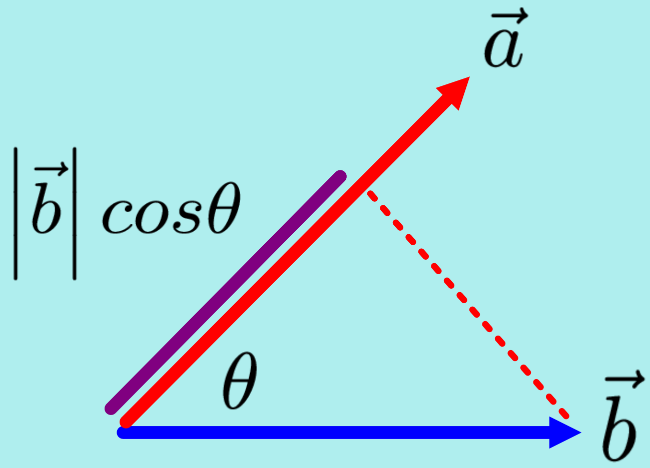
Scalar & Vector Projections

One way to view the dot product is through vector components of one vector onto another. The order of the dot product does not matter.

The relative directions do matter, however, and will cause the dot product to be positive or negative.

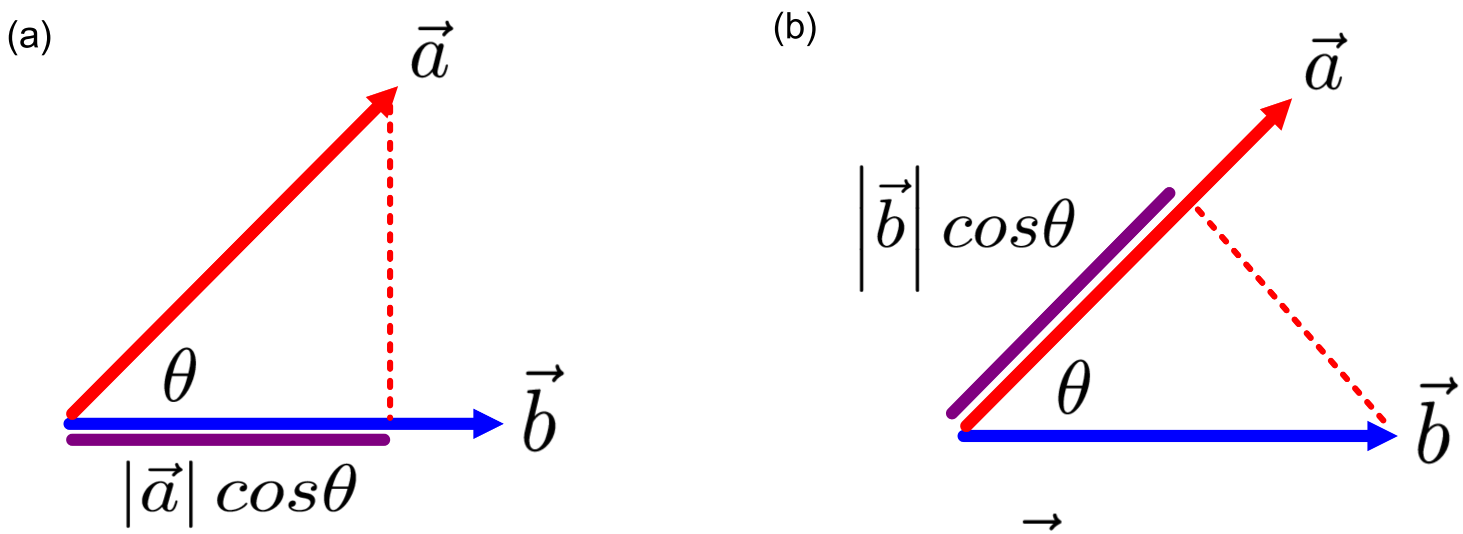




$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

Scalar & Vector Projections

The scalar projection is a measure of how one vector lies along a second vector.



(a) scalar projection of **a** on **b**:
$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = |\vec{a}|cos\theta$$

(b) scalar projection of **b** on **a**:
$$\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = |\vec{b}|cos\theta$$

The vector projection is similar to the scalar projection, but also includes the direction of the second vector.

To include the direction of a vector, but ignore the magnitude, we use the unit vector.

vector projection = scalar projection $\times$ unit vector

vector projection of $\vec{a}$ on $\vec{b}$:

$$\underbrace{\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}}_{\text{scalar projection}} \times \underbrace{\frac{\vec{b}}{|\vec{b}|}}_{\text{unit vector in direction of } \vec{b}} = \frac{(\vec{a}\cdot\vec{b})\,\vec{b}}{|\vec{b}|^2}$$

Projections Using Basis Unit Vectors

Given a point in R^3, $P(a,b,c)$, the vector is

$$\overrightarrow{OP} = (a,b,c)$$

We can consider the scalar and vector projections onto the x-, y-, and z-axes, which are trivial:

	scalar projection	vector projection
x:	a	$a\vec{i}$
y:	b	$b\vec{j}$
z:	c	$c\vec{k}$

From the previous definition of scalar projection:

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$\frac{\overrightarrow{OP} \cdot \vec{i}}{|\vec{i}|} = |\overrightarrow{OP}| cos\alpha = a$$

$$\therefore cos\alpha = \frac{a}{|\overrightarrow{OP}|}$$

"alpha"

Similarly,

$$cos\beta = \frac{b}{|\overrightarrow{OP}|} \quad \text{and} \quad cos\gamma = \frac{c}{|\overrightarrow{OP}|}$$

"beta"

"gamma"

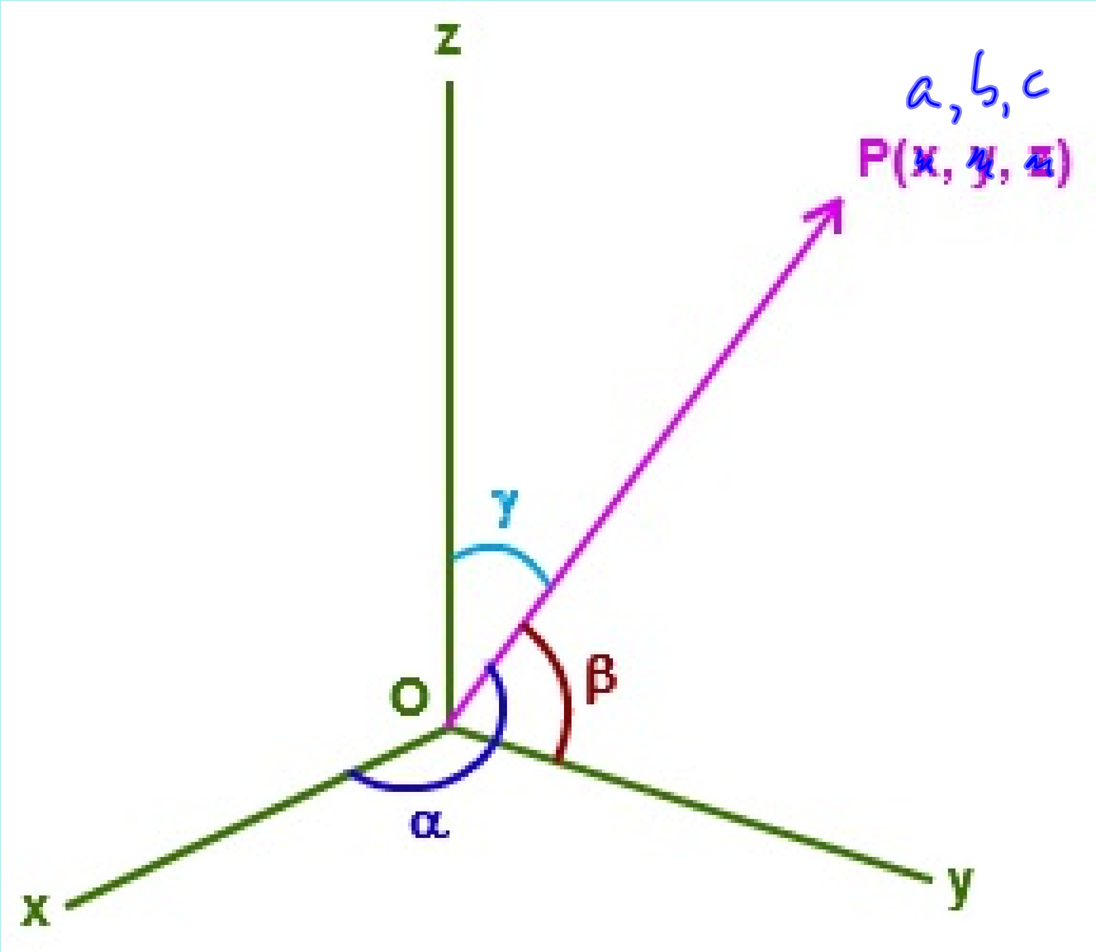
γ

$$\cos\alpha = \frac{a}{\left|\overrightarrow{OP}\right|} \quad \cos\beta = \frac{b}{\left|\overrightarrow{OP}\right|} \quad \cos\gamma = \frac{c}{\left|\overrightarrow{OP}\right|}$$

These are the <u>direction cosines</u> for $\overrightarrow{OP} = (a, b, c)$

where α, β, and γ are the <u>direction angles</u> $\overrightarrow{OP}$ makes with the positive x-, y-, and z-axes.

$\vec{q} = |\vec{q}|$ [direction]



$$\cos\alpha = \frac{a}{\left|\overrightarrow{OP}\right|}$$

$$\cos\beta = \frac{b}{\left|\overrightarrow{OP}\right|}$$

$$\cos\gamma = \frac{c}{\left|\overrightarrow{OP}\right|}$$

Ex.1 For vectors **a** = (-2, 3, 4) and **b** = (8, 7, -6), find:

(a) the scalar projection of **a** onto **b**.

(b) the scalar projection of **b** onto **a**.

(c) the vector projection of **a** onto **b**.

(d) the direction angles for **a**.

$\vec{a}\cdot\vec{b} = (-2)(8) + (3)(7) + (4)(-6)$

$= -19$

$|\vec{a}| = \sqrt{29}$

$|\vec{b}| = \sqrt{149}$

(a) $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$= \dfrac{-19}{\sqrt{149}} \times \dfrac{\sqrt{149}}{\sqrt{149}}$

$= \dfrac{-19\sqrt{149}}{149}$

(b) $\dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \dfrac{-19}{\sqrt{29}} \times \dfrac{\sqrt{29}}{\sqrt{29}}$

$= \dfrac{-19\sqrt{29}}{29}$

$\dfrac{\vec{b}}{|\vec{b}|}$

(c) vect proj $\vec{a}$ on $\vec{b}$ = (scalar proj)(unit vector)

$\vec{a} \downarrow \vec{b} = \left(\dfrac{-19\cancel{\sqrt{149}}}{149}\right)\left(\dfrac{(8,7,-6)}{\cancel{\sqrt{149}}}\right)$

$= \dfrac{-19}{149}(8,7,-6)$

$= \left(\dfrac{-152}{149}, \dfrac{-133}{149}, \dfrac{114}{149}\right)$

(d) $\vec{a} = (-2,3,4)$

$\cos\alpha = \dfrac{x_{\vec{a}}}{|\vec{a}|}$ $\quad\cos\beta = \dfrac{y_{\vec{a}}}{|\vec{a}|}$ $\quad\cos\gamma = \dfrac{4}{\sqrt{29}}$

$\cos\alpha = \dfrac{-2}{\sqrt{29}}$ $\quad = \dfrac{3}{\sqrt{29}}$ $\quad \gamma \doteq 42°$

$\alpha \doteq 112°$ $\quad \beta \doteq 56°$

Assigned Work

p.398 # 1, 6, 7b, 8, 11, 13, 15, 17