Unit 7: Lines & Planes

Equations of Lines in R^2

$y = ax + b$

Recall: equation of a straight line

$$y = mx + b$$

slope, rise/run, rate of change

starting point (y-intercept)

see Geogebra demo

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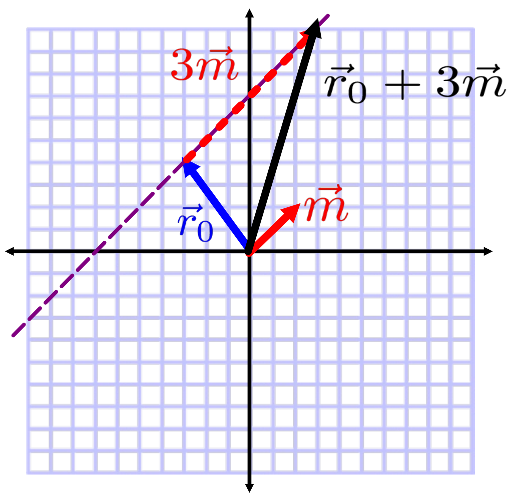
Vector Equation of a Straight Line:

$$\vec{r} = \vec{r}_0 + t\vec{m}$$

vector position of starting point

scale factor $t \epsilon \mathbf{R}$

direction vector (along slope of line)



By varying the value of t, any point along the line can be obtained through vector addition.

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Ex.1 A line passes through the point (5,-2) with the direction vector (4,6).

(a) State the vector equation of the line.

(b) Find the point that corresponds with t = 3.

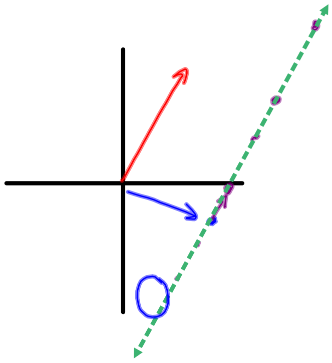
(c) Does the point (1,-6) lie on this line?

(d) Write the equation in the form y = mx + b.

$P(5,-2)$

$\vec{OP} = (5,-2)$

$= \vec{r_0}$



(a) $\vec{r} = \vec{r_0} + t\vec{m}$

$\vec{r} = (5,-2) + t(4,6),\ t \in R$

(b) $\vec{r} = (5,-2) + (3)(4,6)$

$= (5,-2) + (12,18)$

$= (17,16)$

$\therefore$ point is $Q(17,16)$

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Ex.1 A line passes through the point (5,-2) with the direction vector (4,6).

(a) State the vector equation of the line.

(b) Find the point that corresponds with t = 3.

(c) Does the point (1,-6) lie on this line?

(d) Write the equation in the form y = mx + b.

from (a): $\vec{r} = (5,-2) + t(4,6)$

(c) does vector $\vec{r} = (1,-6)$ touch line?

$$\underset{x\quad y}{(1,-6)} = \underset{x\quad y}{(5,-2)} + t\,\underset{x\quad y}{(4,6)}$$

$1 = 5 + 4t$ ① $-6 = -2 + 6t$ ②

$-4 = 4t$

$t = -1$

check $t = -1$ or solve for t

$LS = -6$ $RS = -2 + 6(-1)$ $-4 = 6t$

$= -2 - 6$ $t = \frac{-4}{6}$

$LS \neq RS$ $= -8$ $t = \frac{-2}{3}$

inconsistency

$\therefore$ (1,-6) does NOT lie on the line.

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$\vec{r} = (5, -2) + t(4, 6)$

(d) $y = mx + b$

$y = \frac{3}{2}x + b$

sub $P_0(5, -2)$

$-2 = \frac{3}{2}(5) + b$

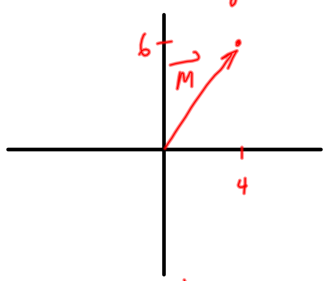
$-2 = \frac{15}{2} + b$

$-\frac{4}{2} - \frac{15}{2} = b$

$b = \frac{-19}{2}$

$$\boxed{y = \frac{3}{2}x - \frac{19}{2}}$$

$\vec{m} = (4, 6)$
$\Delta x \quad \Delta y$



$m = \frac{\Delta y}{\Delta x}$

$= \frac{6}{4}$ *

$= \frac{3}{2}$

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Parametric Equation of a Straight Line:

The parametric form of the equation comes directly from the vector equation. It considers the x-, and y-components separately, which can be more convenient.

$$\vec{r} = \vec{r}_0 + t\vec{m}$$
$$(x, y) = (x_0, y_0) + t(a, b)$$

$$\boxed{\begin{aligned} \Rightarrow x &= x_0 + at \\ \Rightarrow y &= y_0 + bt \end{aligned}}$$

2 parametric eqns in R^2

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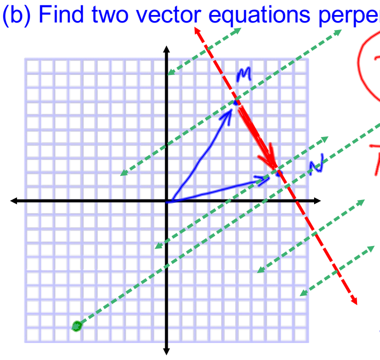
Ex.2

(a) Find the vector and parametric equations of the line passing through M (5,7) and N (8,2).

(b) Find two vector equations perpendicular to your answer in part (a).



$\vec{m} = \overrightarrow{MN}$ or $\vec{m} = \overrightarrow{NM}$

$\vec{m} = \overrightarrow{ON} - \overrightarrow{OM}$

$= (8,2) - (5,7)$

$= (3,-5)$

$\vec{r} = \vec{r}_0 + t\vec{m}$

$\overrightarrow{OM}$ or $\overrightarrow{ON}$

vector

$$\boxed{\vec{r} = (5,7) + t(3,-5)}$$

$(x,y) = (5,7) + t(3,-5)$

$$\boxed{\begin{aligned} x &= 5 + 3t \\ y &= 7 - 5t \end{aligned}}$$ parametric

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Ex.2

(a) Find the vector and parametric equations of the line passing through M (5,7) and N (8,2).

(b) Find two vector equations perpendicular to your answer in part (a).

from (a): $\vec{r} = (5,7) + t(\underset{\Delta x}{3}, \underset{\Delta y}{-5})$

slope

$m = \frac{\Delta y}{\Delta x}$

$= \frac{-5}{3}$

$\Rightarrow m_\perp = \frac{3}{5}$ $\frac{\Delta y}{\Delta x}$

$\Rightarrow \vec{m}_\perp = (\underset{\Delta x}{5}, \underset{\Delta y}{3})$

$\vec{r}_\perp = (5,7) + t(5,3)$

same starting vector!

OR

$\vec{r}_\perp = (8,2) + t(5,3)$

OR

in general

$\vec{r}_\perp = \underbrace{(a,b)}_{\text{any start is valid}} + t\underbrace{(5,3)}_{\text{direction vector must be } \perp}$

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Assigned Work:

p.433 # 1, 2, 3, 4, 5, 6, 7, 9b, 10

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