Equations of a Line in R3

(1) Slope-Intercept: $y = mx + b$

No equivalent form in R3.

(2) Vector form: $\vec{r} = \vec{r}_0 + t\vec{m}$ $t \in \mathbb{R}$

Each vector now has three components.

$$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$$

(3) Parametric form:

$$x = x_0 + ta \qquad y = y_0 + tb \qquad z = z_0 + tc$$

(4) Cartesian form: $Ax + By + Cz + D = 0$

In R3, this form represents a plane, not a line.

$Ax + By + C = 0$ in R^2

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(5) Symmetric form:

The parametric equations all have a common factor 't'.

$$x = x_0 + ta \qquad y = y_0 + tb \qquad z = z_0 + tc$$

Rearranging each equation for 't' yields:

$$t = \frac{x - x_0}{a} \qquad t = \frac{y - y_0}{b} \qquad t = \frac{z - z_0}{c}$$

Since the values of 't' are all the same,

$$\boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}} \qquad a, b, c \neq 0$$

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Ex.1 Determine vector, parametric, and symmetric equations for the line passing through P(-2, 3, 5) and Q(-2, 4, -1).

in R^2: $m = \frac{\Delta y}{\Delta x}$ $\vec{m} = (\Delta x, \Delta y)$

(a) $\vec{r} = \vec{r}_0 + t\vec{m}$, $t \in R$

$\vec{r}_0 = \vec{OP}$ or $\vec{OQ}$
$= (-2, 3, 5)$

$\vec{m} = (\Delta x, \Delta y, \Delta z)$
$= \vec{PQ}$
$= \vec{OQ} - \vec{OP}$
$= (-2-(-2), 4-3, -1-5)$
$= (0, 1, -6)$

$(x, y, z) = (-2, 3, 5) + t(0, 1, -6)$

(b) $x = -2 + 0t$ $\quad y = 3 + t$ $\quad z = 5 - 6t$

(c) $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$ $\quad a, b, c \neq 0$
$\vec{m} = (a, b, c)$

$x = -2$ $\quad y = 3 + t$ $\quad z = 5 - 6t$
$y - 3 = t$ $\quad +z - 5 = -6t$
$\frac{z-5}{-6} = t$

$x = -2$ $\quad \frac{y-3}{1} = \frac{+z-5}{-6}$

$\vec{m} = (0, 1, -6)$

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Ex.2 Given the symmetric equation, determine vector and parametric equations.

$$\frac{x-3}{5} = \frac{y+2}{3} = \frac{z+5}{7}$$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

$\vec{m} = (5, 3, 7)$ $\quad \vec{r}_0 = (3, -2, -5)$

$\vec{r} = (3, -2, -5) + t(5, 3, 7)$ $\quad t \in R$

$x = 3 + 5t$ $\quad y = -2 + 3t$ $\quad z = -5 + 7t$

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Assigned Work:

p.449 # 3, 6, 7, 8, 9, 12, 14

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