Vector & Parametric Equation of a Plane

Equations of a Line in R^3:

vector:
$$\vec{r} = \vec{r}_0 + t\vec{m}$$
$$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$$

parametric:
$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

symmetric:
$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

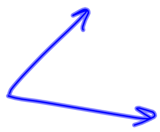
May 23-8:30 AM

Vector & Parametric Equation of a Plane

A plane can be described using a point and two vectors.
The vector equation of a plane is written:

$$\vec{r} = \vec{r}_0 + m\vec{d}_1 + n\vec{d}_2$$

$m \in \mathbb{R}$
$n \in \mathbb{R}$



$$(x, y, z) = (x_0, y_0, z_0) + m(a_1, b_1, c_1) + n(a_2, b_2, c_2)$$

where: $\vec{r}$ is the position vector of any point in the plane

$\vec{r}_0$ is the position vector of a starting point

$\vec{d}_1$ and $\vec{d}_2$ are direction vectors for the plane (they must be noncollinear)

m and n are parameters (real numbers)

May 16-11:39 AM

Vector equation of a plane:

$$\vec{r} = \vec{r}_0 + m\vec{d}_1 + n\vec{d}_2$$

$$(x, y, z) = (x_0, y_0, z_0) + m(a_1, b_1, c_1) + n(a_2, b_2, c_2)$$

In parametric form:

$$x = x_0 + ma_1 + na_2$$

$$y = y_0 + mb_1 + nb_2$$

$$z = z_0 + mc_1 + nc_2$$

May 16-1:19 PM

Ex.1 Find the vector equation of the plane containing the points L(1,2,5), M(-7,4,0) and N(3,1,-2).

$$\vec{r} = \vec{r}_0 + \underbrace{m\vec{d}_1 + n\vec{d}_2}$$

$\vec{r}_0$: $\overrightarrow{OL}$ or $\overrightarrow{OM}$ or $\overrightarrow{ON}$

need 2 vectors within plane.

$\overrightarrow{LM}$, $\overrightarrow{MN}$, $\overrightarrow{LN}$ } pick 2

e.g., $\vec{r} = \overrightarrow{OL} + m\overrightarrow{LM} + n\overrightarrow{MN}$

$\vec{r} = (1,2,5) + m(-8,2,-5) + n(10,-3,-2)$

$\overrightarrow{LM} = \overrightarrow{OM} - \overrightarrow{OL}$
$= (-8,2,-5)$

$\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM}$
$= (10,-3,-2)$

May 19-4:00 PM

Ex.2 Does the point (4,5,-3) lie in the plane

x = 4 + 3g - 6h, y = 1 - 2g + 6h, z = 6 + g - h

let $P(4,5,-3) \Rightarrow \vec{OP} = (4,5,-3)$
x y z

test $\vec{OP}$ in parametric eqns.

$4 = 4 + 3g - 6h$ ① $5 = 1 - 2g + 6h$ ② $-3 = 6 + g - h$ ③

$0 = 3g - 6h$
$4 = -2g + 6h$
$4 = g$

$\underbrace{4 = -2g + 6h}$

LS = 4

RS = $-2(4) + 6(13)$
$= -8 + 78$
$= 70$

$-9 = g - h$
$-9 = 4 - h$
$h = 13$

LS ≠ RS

∴ (4,5,-3) does <u>not</u> lie in plane.

May 16-11:48 AM

Ex.3 Where does the plane π intersect with the line L.

π: $\vec{r} = (6,-2,-3) + m(1,3,0) + n(2,2,-1)$

L: $\vec{r} = t(0,1,0)$
$= (0,0,0) + t(0,1,0)$

want $\vec{r}_\pi = \vec{r}_L$

$(6,-2,-3) + m(1,3,0) + n(2,2,-1) = t(0,1,0)$

$6 + m + 2n = 0$ ① $-2 + 3m + 2n = t$ ② $-3 - n = 0$ ③

$6 + m + 2(-3) = 0$
$6 + m - 6 = 0$
$m = 0$
?

$-2 + 3(0) + 2(-3) = t$
$-2 - 6 = t$
$t = -8$

$-3 = n$

L: $\vec{r} = t(0,1,0)$
$= -8(0,1,0)$
$= (0,-8,0)$

∴ PoI is
P(0,-8,0)

May 31-1:40 PM

Assigned Work:

p.459-460 #1, 2, 3, 4, 6, 8a,
9, 10, 11, 12b

Apr 26-4:51 PM