Intersection of Lines in R^3

Recall: Intersection of Lines in R^2
(i.e., systems of linear equations)

For two lines in R2, there are three possible outcomes:

(1) One solution (lines intersect at a point).

(2) No solution (lines are parallel, no intersection).

(3) Infinite solutions (lines are coincident).

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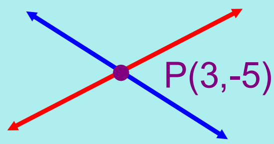
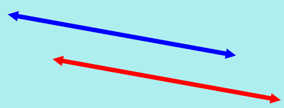
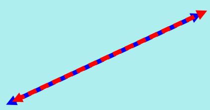
outcome in R^2	algebra	sketch
one solution	result makes sense (e.g., $x = 3$, $y = -5$)	
no solution	result not possible (e.g., $0 = 1$)	
infinite solutions	result always true (e.g., $3 = 3$)	

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Intersection of Lines in R^3

A linear system may have:

A) One unique solution:
- the lines intersect at one point
- the angle between two lines may be calculated from the dot product of direction vectors

B) No solution:
- the lines do not intersect
- the lines are parallel & distinct (i.e., in the same plane)

OR

- the lines may be skew (i.e., they are not parallel)
- they do not intersect because they lie in different planes

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B) No solution:
- the lines do not intersect
- the lines are parallel & distinct (i.e., in the same plane)

OR

- the lines may be skew (i.e., they are not parallel)
- they do not intersect because they lie in different planes

C) Infinite solutions:
- the lines are coincident

A linear system of two (or more) equations is said to be <u>consistent</u> if it has <u>at least</u> one solution, otherwise it is <u>inconsistent</u> (no solutions).

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Equations of Lines in R^2:

slope-intercept	$y = mx + b$
cartesian	$Ax + By + C = 0$
vector	$\vec{r} = \vec{r}_0 + t\vec{m} \qquad t \in \mathbb{R}$
parametric	$x = x_0 + tm_x$ $y = y_0 + tm_y \qquad t \in \mathbb{R}$

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Equations of Lines in R^3:

vector	$\vec{r} = \vec{r}_0 + t\vec{m} \qquad t \in \mathbb{R}$
parametric	$x = x_0 + tm_x$ $y = y_0 + tm_y \qquad t \in \mathbb{R}$ $z = z_0 + tm_z$
symmetric	$\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$ $a, b, c \neq 0$

$\vec{d} = (a, b, c)$

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Ex.1 Solve the following systems and describe the geometric relationship between the lines.

(a) $\vec{r}_1 = (-2, 0, -3) + t(5, 1, 3)$ $t \in \mathbb{R}$
$\vec{r}_2 = (5, 8, -6) + s(-1, 2, -3)$ $s \in \mathbb{R}$

$\vec{r}_1$

$x_1 = -2 + 5t$ 2
$y_1 = 0 + t$ 2
$z_1 = -3 + 3t$ 2

$\vec{r}_2$

$x_2 = 5 - s$
$y_2 = 8 + 2s$
$z_2 = -6 - 3s$

Set $x_1 = x_2$
$y_1 = y_2$
$z_1 = z_2$

$-2 + 5t = 5 - s$ ① $t = 8 + 2s$ ② $-3 + 3t = -6 - 3s$ ③

①×2 : $-4 + 10t = 10 - 2s$

$-4 + 11t = 18$
$11t = 22$
$t = 2$

Sub $t = 2$ into ②

$2 = 8 + 2s$
$-6 = 2s$
$s = -3$

Verify ③ $LS = -3 + 3t$
$= -3 + 3(2)$
$= -3 + 6$
$= 3$

$RS = -6 - 3s$
$= -6 - 3(-3)$
$= -6 + 9$
$= 3$

$LS = RS$

∴ system is consistent.

∴ solution is a point $P(8, 2, 3)$

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(b) $\dfrac{x+4}{3} = \dfrac{y-12}{4} = \dfrac{z-3}{6}$ l_1

$P_1(-4, 12, 3)$

$\dfrac{x}{\frac{1}{2}} = \dfrac{y-10}{\frac{2}{3}} = \dfrac{z+5}{1}$ l_2

$P_2(0, 10, -5)$

collinear
→ no sol'n
→ infinite

(1) Brute force → convert to parametric
→ solve.

(2) recall : $\vec{d}_1 = (3, 4, 6)$ $\vec{d}_2 = \left(\frac{1}{2}, \frac{2}{3}, 1\right)$

collinear? $\vec{d}_1 = k\vec{d}_2$

$3 = k\left(\frac{1}{2}\right)$ $4 = k\left(\frac{2}{3}\right)$ $6 = k(1)$

$k = 6.$

$LS = 3$
$RS = 6\left(\frac{1}{2}\right)$
$= 3$

$LS = 4$
$RS = 6\left(\frac{2}{3}\right)$
$= 4$

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(b) $\frac{x+4}{3}=\frac{y-12}{4}=\frac{z-3}{6}$

$P_1(-4,12,3)$

$\frac{x}{\frac{1}{2}}=\frac{y-10}{\frac{2}{3}}=\frac{z+5}{1}$

$P_2(0,10,-5)$

test $P_2(0,10,-5)$ on ℓ_1

$\frac{x+4}{3}=\frac{0+4}{3}=\frac{4}{3}$

$\frac{y-12}{4}=\frac{10-12}{4}=\frac{-2}{4}=\frac{-1}{2}$

$\frac{z-3}{6}=\frac{-5-3}{6}=\frac{-8}{6}=\frac{-4}{3}$

$\because$ inconsistent results

$\therefore$ P_2 is not on ℓ_1

$\therefore$ no solution (lines are parallel but distinct)

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Assigned Work:

p.497 # 8, 9, 11, 12

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