Intersections of a Line and a Plane (in $\mathbb{R}^3$)

Three possibilities:

(a) no solution	The line and the plane are parallel and distinct; no point of intersection.
(b) one solution	The line passes through the plane; one point of intersection.
(c) infinite solutions	The line is in the plane; infinite points of intersection (i.e., the solution is the entire line).

May 16-11:39 AM

Ex.1 Find the intersection of the line and the plane:

$\ell: \quad \vec{r} = (4, 6, -2) + t(-1, 2, 1) \qquad t \in \mathbb{R}$

$\pi: \quad 2x - y + 6z + 10 = 0$

$x = 4 - t$

$y = 6 + 2t$

$z = -2 + t$

$2(4-t) - (6+2t) + 6(-2+t) + 10 = 0$

$8 - 2t - 6 - 2t - 12 + 6t + 10 = 0$

$0 + 2t = 0$

$t = 0$

to find PoI, set $t = 0$:

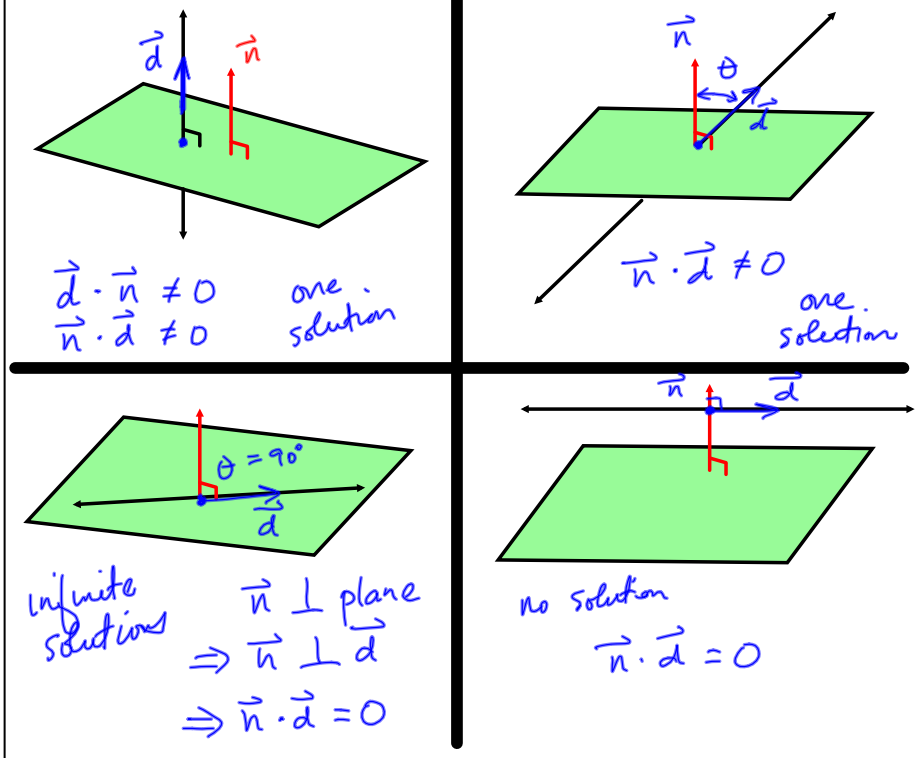
$\vec{r} = (4,6,-2) + (0)(-1,2,1)$

$= (4,6,-2)$

$\therefore$ PoI is $P(4,6,-2)$

May 19-4:00 PM

Compare the direction vector of the line to the normal of the plane.
Can we draw any conclusions?



Jun 6-2:59 PM

Ex.2 Given the equation of a line and plane:
(a) Is there any way to predict the number of solutions?
(b) Determine the intersection (if any).

$\vec{n} \cdot \vec{d}$?

ℓ

$$x = 5 + t$$
$$y = 4 + 2t$$
$$z = 7 + 2t$$

$\vec{d} = (1, 2, 2)$

π

$$2x + 3y - 4z - 7 = 0$$

$\vec{n} = (2, 3, -4)$

$\vec{n} \cdot \vec{d} = 2(1) + 3(2) + (-4)(2)$
$= 2 + 6 - 8$
$= 0$ $(\theta = 90°)$
$\vec{n} \perp \vec{d}$
$\therefore$ infinite or no solution

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Ex.2 Given the equation of a line and plane:
(a) Is there any way to predict the number of solutions?
(b) Determine the intersection (if any).

$x = 5 + t$
$y = 4 + 2t$
$z = 7 + 2t$

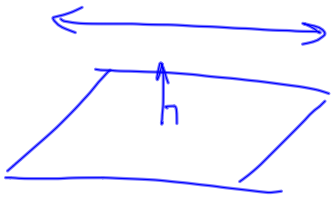
$2x + 3y - 4z - 7 = 0$

$2(5+t) + 3(4+2t) - 4(7+2t) - 7 = 0$
$10 + 2t + 12 + 6t - 28 - 8t - 7 = 0$
$-13 + 0t = 0$

not possible $\begin{cases} -13 = 0 \\ 0 = 13 \end{cases}$

inconsistent system

$\therefore$ no solution



line parallel to plane (but distinct)

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Assigned Work:

p.496 # 3, 4, 5, 6, 7, 13

Apr 26-4:51 PM