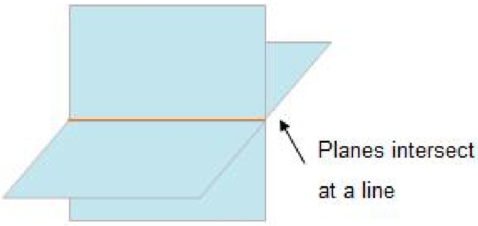
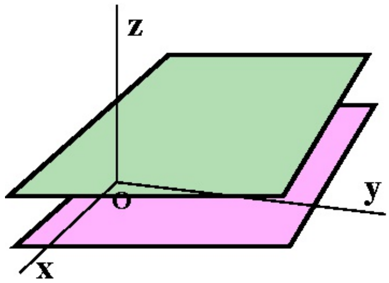
Intersections of Two Planes

(1) If two planes intersect along a line, the system has an infinite number of solutions, as described by the parametric equations of the line.



(2) If two planes are coincident (i.e., same plane), the system has an infinite number of solutions, as described by either of the two given equations of the plane.

(3) If two planes are parallel (i.e., their normal vectors are parallel) and distinct, the system has no solution.



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Ex.1 Solve each system and give a geometric description of the planes.
(i.e., line ~~intersection~~, coincident, parallel & distinct)

a) $x + 4y - 3z + 6 = 0$ ①
$2x + 8y - 6z + 11 = 0$ ②

$\vec{n}_1 = (1, 4, -3)$
$\vec{n}_2 = (2, 8, -6)$

$\frac{2}{1} = 2 \quad \frac{8}{4} = 2 \quad \frac{-6}{-3} = 2$

$\vec{n}_2 = 2\vec{n}_1$

① ×2 : $2x + 8y - 6z + 12 = 0$
$0 + 0 - 0 - 1 = 0$
$-1 = 0$
not possible

inconsistent ∴ no solution
parallel but distinct

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b) $5x - y + 2z - 9 = 0$ ①
$-25x + 5y - 10z + 45 = 0$ ②

$\vec{n}_1 = (5, -1, 2)$
$\vec{n}_2 = (-25, 5, -10)$

$\vec{n}_1 = k\vec{n}_2$?

$5 = k(-25)$
$k = \frac{-1}{5}$

$-1 = k(5)$
$k = \frac{-1}{5}$

$2 = k(-10)$
$k = \frac{-1}{5}$

① ×5: $25x - 5y + 10z - 45 = 0$

$0 + 0 + 0 + 0 = 0$

$0 = 0$

always true

∴ infinite solutions

Same plane

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c) $4x + 7y - 33z + 17 = 0$ ①
$8x + 5y - 3z + 7 = 0$ ②

$\vec{n}_1 = (4, 7, -33)$
$\vec{n}_2 = (8, 5, -3)$

planes not parallel
solution is line

① ×2: $8x + 14y - 66z + 34 = 0$

$\frac{-9y + 63z - 27}{-9} = \frac{0}{-9}$

$y - 7z + 3 = 0$

$y = 7z - 3$ let $z = t$

$y = 7t - 3$

Sub z and y into ①

$4x + 7(7t - 3) - 33t + 17 = 0$

$4x + 49t - 21 - 33t + 17 = 0$

$4x + 16t - 4 = 0$

$4x = -16t + 4$

$$\boxed{\begin{aligned} x &= -4t + 1 \\ y &= 7t - 3 \\ z &= t + 0 \end{aligned}} \quad t \in R$$

$\vec{r} = (1, -3, 0) + t(-4, 7, 1)$

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Assigned Work

p.516 # 1, 2, 3, 6, 8, 10

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