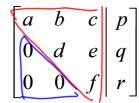


To find solutions to the system, we continue to manipulate the matrix until it is in <u>row-echelon form</u>:



notice the upper-triangle of non-zero values, and the lower-triangle of zeroes

Continuing with our previous example, row 1 and row 2 are already in row-echelon form:

$\begin{bmatrix} 1 & 2 \end{bmatrix}$	2 9]	[1	2	2	9 -]
0 -1	$ \begin{bmatrix} 2 & 9 \\ -2 & -8 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -2r_1 + r_3 \\ -2r_2 + r_3 \\ -2r_1 + $	0	- (-2	- 8	
2 3	-1 1	6	-1	-5	-17	
		r		- 1	2	
0 0	-r2+f3	(2	2	9	
	-r2+r3	0	-1	-2	-8	
		0	ò	-3	-9)

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The row-echelon augmented matrix is equivalent to a system of new, but equivalent, equations:

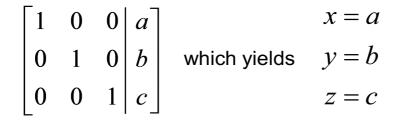
$$\begin{bmatrix} 1 & 2 & 2 & | & 9 \\ 0 & -1 & -2 & | & -8 \\ 0 & 0 & -3 & | & -9 \end{bmatrix} \implies x + 2y + 2z = 9$$

$$\Leftrightarrow \qquad -y - 2z = -8 \\ -3z = -9$$

This system is easily solved for z, and the rest can be solved by back-substitution. This technique is called <u>Gaussian Elimination</u>.

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It is also possible to find solutions directly from the matrix by continuing to work towards a <u>reduced row-echelon form</u>. Solving this way is called <u>Gauss-Jordan elimination</u>.



Ex.1 Write in reduced row-echelon form and solve

$$\begin{bmatrix} 1 & 2 & 2 & 9 \\ 0 & -1 & -2 & -8 \\ 0 & 0 & -3 & -9 \end{bmatrix}$$

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$$\begin{array}{c} \text{(continued...)}\\ \text{Ex.1 Write in reduced row-echelon form and solve} \\ \begin{bmatrix} 1 & 2 & 2 & | & 9 \\ 0 & -1 & -2 & | & -8 \\ 0 & 0 & -3 & | & -9 \end{bmatrix} \xrightarrow{-r_{-}} \begin{bmatrix} 1 & 2 & 2 & | & 9 \\ 0 & 1 & 2 & | & 8 \\ 0 & 0 & -3 & | & -9 \end{bmatrix} \xrightarrow{-r_{-}} \begin{bmatrix} 1 & 2 & 2 & | & 9 \\ 0 & 1 & 2 & | & 8 \\ 0 & 0 & 1 & | & 2 \end{bmatrix} \\ \xrightarrow{-2r_{+}+r_{+}} \begin{bmatrix} 1 & (2) & 2 & | & 9 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & (& | & 3 \end{bmatrix} \\ \xrightarrow{-2r_{+}+r_{+}} \begin{bmatrix} 1 & (2) & 2 & | & 9 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & (& | & 3 \end{bmatrix} \\ \xrightarrow{-2r_{+}+r_{+}} \begin{bmatrix} 1 & 0 & (2) & | & 5 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \\ \xrightarrow{-2r_{+}+r_{+}} \begin{bmatrix} 1 & 0 & (2) & | & 5 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \\ \xrightarrow{-2r_{+}+r_{+}} \begin{bmatrix} 1 & 0 & (2) & | & 5 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & (& | & 3 \end{bmatrix} \\ \xrightarrow{\chi} = -1 \\ \begin{array}{c} \chi = -1 \\ \chi = 2 \\ \chi = 3 \end{bmatrix} \end{array}$$

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Assigned Work:

p.588 # 4, 6b, 8b, 10ace p.594 # 1c, 3ace

read (diagrams & text) on p.520-521 p.526-527

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