Distances Between Points, Lines, and Planes

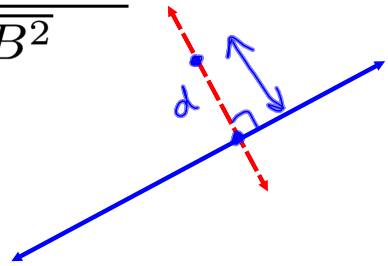
1. Distance from a Point to a Line in R^2

Given the point, $P(x_0, y_0)$

and a line in Cartesian form, $L : Ax + By + C = 0$

The distance from P to L is given by

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$



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2. Distance from a Point to a Line in R^3

In R^3, there is no Cartesian equation for a line, so the vector equation must be used:

$$L : \vec{r} = \vec{r}_0 + t\vec{m}$$

The distance from a point, P, to the line, L, is given by

$$d = \frac{\left|\vec{m} \times \overrightarrow{QP}\right|}{|\vec{m}|}$$

where Q is a known point on the line, $\vec{r}_0 = \overrightarrow{OQ}$

$\vec{m}$ is the direction vector for the line

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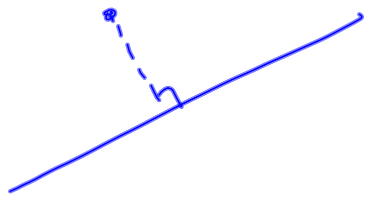
3. Distance from a Point to a Plane

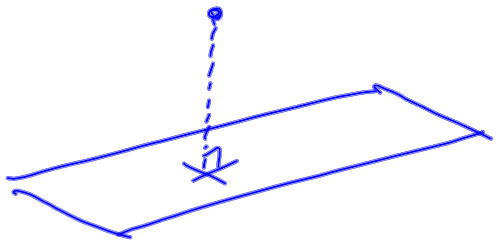
$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

where the plane is defined by the Cartesian equation

$$Ax + By + Cz + D = 0$$

and the point is $P(x_0, y_0, z_0)$





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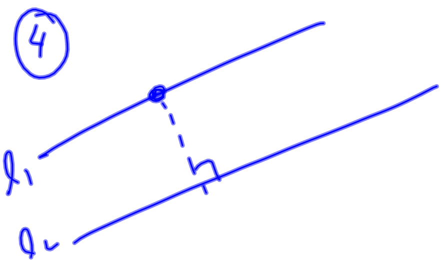
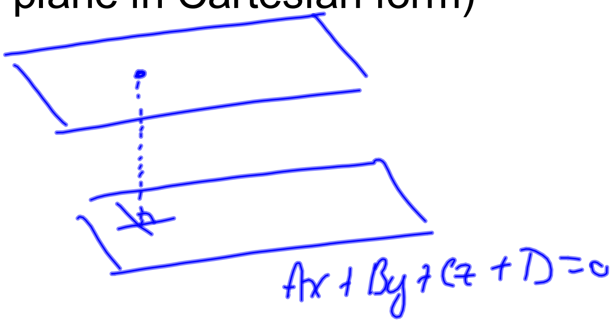
Other cases:

4. In R^2 or R^3, distance between two parallel lines:
   (1) Find a point on one line using equation.
   (2) Find distance between point and other line.

5. In R^3, distance between two parallel planes.
   (1) Find a point on one plane using equation.
   (2) Find distance between point and other plane.
   (note: need 2nd plane in Cartesian form)





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Other cases:

6. In R^3, distance between two skew lines.

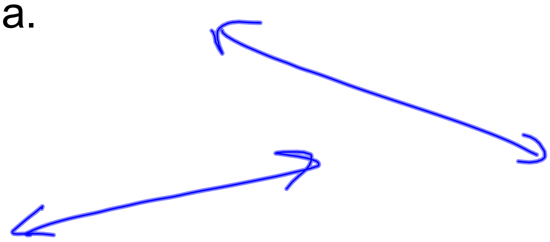
(1) Use both direction vectors to create one plane at position vector of first line.

(2) Use both direction vectors to create 2nd plane at position vector of second line.

Both planes are parallel

(3) Determine a point on first plane.

(4) Convert second plane to Cartesian form and apply formula.



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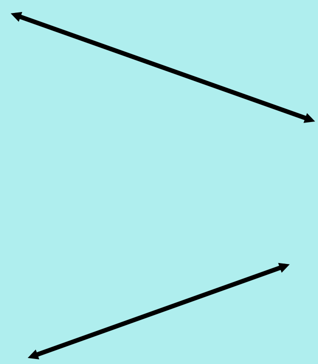
Case 6: Two skew lines in R^3



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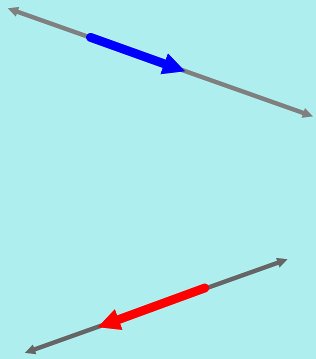
(1) Given (or determine) direction vectors for each line:



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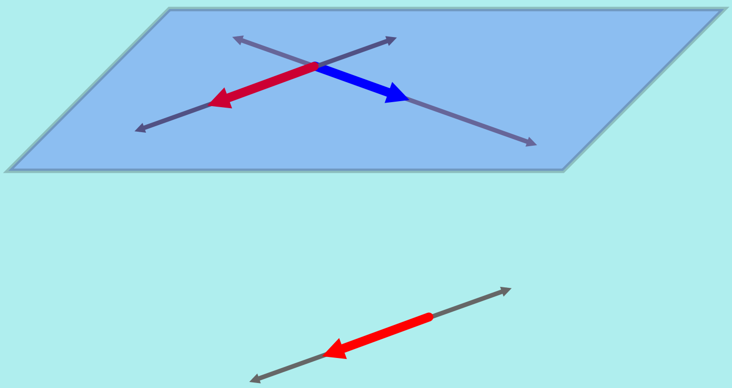
(2) Use 2nd direction vector (red), combined with 1st direction vector (blue) to make a plane.



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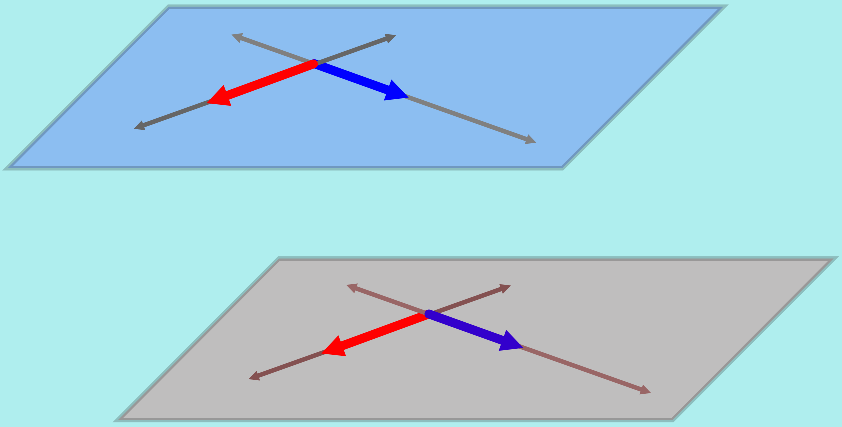
(3) Create a 2nd plane using both vectors.



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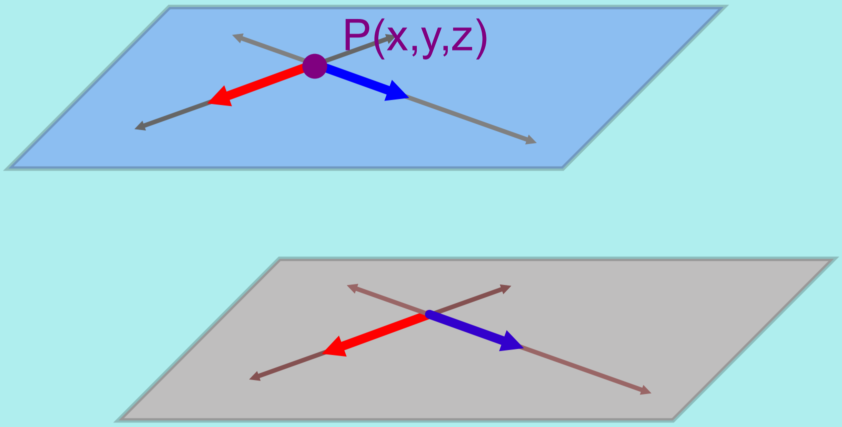
(4) Find a point on one of the planes (use initial position vector from first line if given).



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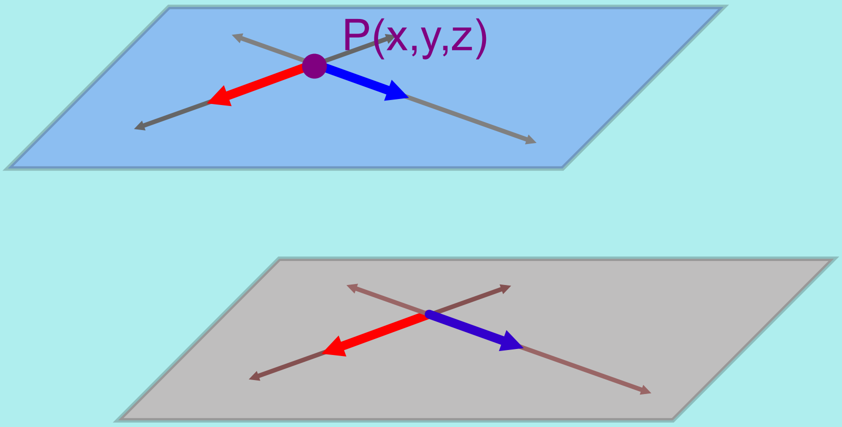
(5) Determine Cartesian equation for other plane
(from vector/parametric equation you created).



Ax + By + Cz + D = 0

Recall: $\vec{n} = (A, B, C) = \vec{d_1} \times \vec{d_2}$

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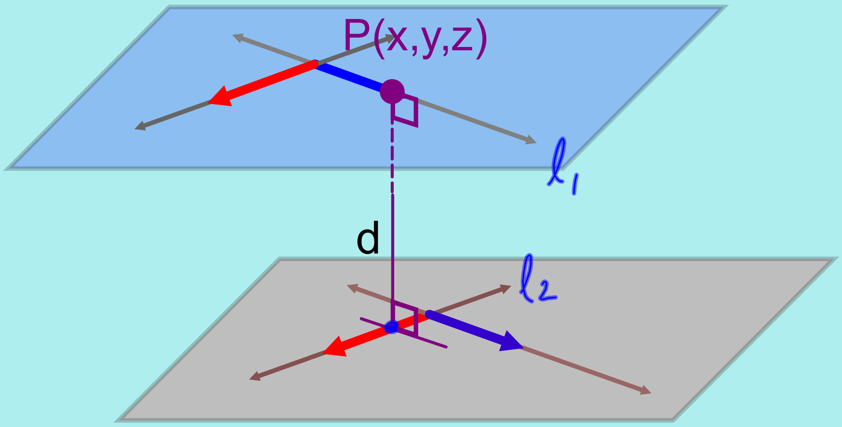
(6) Use point and Cartesian equation to determine
(perpendicular) distance between planes.



Ax + By + Cz + D = 0

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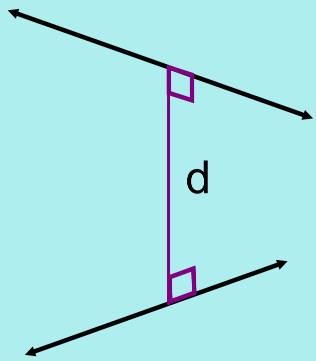
Note: This is also the shortest (perpendicular) distance between the two original lines.



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Assigned Work:

p.540 # 2a, 3a, 5ac, 6b, 8
p.550 # 2a, 3a, 5, 6

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