Solving Mathematical Problems Using Computer Science

Caley Contest (Grade 10)

1. (2009) How many integers n are there with the property that the product of the digits of n is 0, where $5000 \le n \le 6000$?

   (A) 332 (B) 270 (C) 301 (D) 272 (E) 299

2. (2009) A "double-single" number is a three-digit number made up of two identical digits followed by a different digit. For example, 553 is a double-single number. How many double-single numbers are there between 100 and 1000?

   (A) 81 (B) 18 (C) 72 (D) 64 (E) 90

Fermat Contest (Grade 11)

1. (2009) How many four-digit integers between 5000 and 6000 are there for which the thousands digit equals the sum of the other three digits? (The thousands digit of 5124 is 5.)

2. (2007) How many three-digit positive integers x are there with the property that x and 2x have only even digits? (One such number is x = 420, since 2x = 840 and each of x and 2x has only even digits.)

3. (2005) The sum of the first 2005 terms of the sequence 1, 2, 3, 4, 1, 2, 3, 4, . . . is _____.

4. (2005) The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is

   (A) 18 (B) 19 (C) 23 (D) 25 (E) 22

5. (2002) How many three-digit positive integers are perfect squares?

   (A) 23 (B) 22 (C) 21 (D) 20 (E) 19

Euclid Contest (Grade 12)

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

1. (2008 #3) A 3 × 3 square frame is placed on a grid of numbers, as shown. In the example, the sum of the numbers inside the square frame is 108, and the middle number is 12. When the square frame is moved to a new position, the sum of its numbers becomes 279. In the frame's new position, what is the middle number?

2. (2007 #3) The first term of a sequence is 2007. Each term, starting with the second, is the sum of the cubes of the digits of the previous term. What is the 2007th term?

3. (2006 #2a) The average of the digits of the integer 46 is 5. Including 46, how many two-digit positive integers have the average of their digits equal to 5?

4. (2003 #4b) Determine the number of positive integers less than or equal to 300 that are multiples of 3 or 5, but are not multiples of 10 or 15.

5. (2003 #5b) A two-digit number has the property that the square of its tens digit plus ten times its units digit equals the square of its units digit plus ten times its tens digit. Determine all two-digit numbers which have this property, and are prime numbers.

Sun Life Financial (Open)

1. (2007 #5) The numbers 1, 2, 3, . . . , 9 are placed in a square array. The sum of the three rows, the sum of the three columns, and the sum of the two diagonals are added together to form a "grand sum", S. For example, if the numbers are placed as shown, the grand sum is:

1	2	3
4	5	6
7	8	9

$$\begin{aligned} S &= \text{row sums} + \text{column sums} + \text{diagonal sums} \\ &= 45 + 45 + 30 \\ &= 120 \end{aligned}$$

What is the maximum possible value of the grand sum S?

2. (2006 #6) Determine the number of integers n that satisfy all three of the conditions below:
   • each digit of n is either 1 or 0,
   • n is divisible by 6, and
   • $0 < n < 10^7$

3. (2004 #2) Let S be the set of all three-digit positive integers whose digits are 3, 5 and 7, with no digit repeated in the same integer. Calculate the remainder when the sum of all of the integers in S is divided by 9.

4. (2003 #8) Determine the number of triples (k, l, m) of positive integers such that

$$k + l + m = 97 \text{ and } \frac{4k}{5} + \frac{5l}{6} + \frac{6m}{7} = 82$$

5. (2002 #7) N is a five-digit positive integer. A six-digit integer P is constructed by placing a 1 at the right-hand end of N. A second six-digit integer Q is constructed by placing a 1 at the left-hand end of N. If P is three times Q, determine the value of N.

6. (2000 #5) How many five-digit positive integers have the property that the product of their digits is 2000?