Factoring Sum or Difference Patterns

Some patterns are so useful that they are worth memorizing. They allow you to work more efficiently and sometimes simplify a much more complicated looking problem.

Recall: Perfect Squares & Difference of Squares

$$\left.\begin{aligned} a^2 + 2ab + b^2 &= (a+b)^2 \\ a^2 - 2ab + b^2 &= (a-b)^2 \end{aligned}\right\} \text{perfect squares}$$

$$\underbrace{a^2 - b^2 =}_{\text{diff. squares}} (a-b)(a+b)$$

Recall: Pascal's Triangle and $(a+b)^n$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$a^0 = 1$

$$\begin{array}{c|lcc} n & & & \\ 0 & (a+b)^0 & = & 1 \\ 1 & (a+b)^1 & = & a+b \\ 2 & (a+b)^2 & = & 1a^2 + 2ab + b^2 \\ 3 & (a+b)^3 & = & 1a^3 + 3a^2b + 3ab^2 + 1b^3 \\ 4 & (a+b)^4 & = & a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \end{array}$$

For $(a-b)^n$, negative coefficients for odd powers of b.

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

Sum & Difference of Cubes:

Ex. Consider $f(x) = x^3 - 27$

factor theorem

(a) What value of x makes f(x) = 0 ?
(b) Factor using long or synthetic division.

(a) $f(3) = (3)^3 - 27$
$= 27 - 27$
$= 0$

$x - 3$ is a factor

(b)
$$\begin{array}{r|l} & x^2 + 3x + 9 \\ \hline x-3 & x^3 + 0x^2 + 0x - 27 \\ & \underline{x^3 - 3x^2} \\ & \quad 3x^2 + 0x \\ & \quad \underline{3x^2 - 9x} \\ & \qquad 9x - 27 \\ & \qquad \underline{9x - 27} \\ & \qquad\quad 0 \end{array}$$

$x^3 - 27 = (x-3)(x^2+3x+9)$ ✓

S 3
P 9
I ✗

$D = b^2 - 4ac$
$= (3)^2 - 4(1)(9)$
$= 9 - 36$
$< 0 \longrightarrow$ no solutions (cannot be factored)

In general,

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Note: the quadratic factor cannot be factored further.

With higher-order polynomials, look for patterns involving squares or cubes.

For example,

$$a^4 - b^4 = \left(a^2\right)^2 - \left(b^2\right)^2$$

$$\begin{aligned} x^{12} + y^{12} &= \left(x^3\right)^4 + \left(y^3\right)^4 \\ &= \left(x^6\right)^2 + \left(y^6\right)^2 \\ &= \left(x^4\right)^3 + \left(y^4\right)^3 \end{aligned}$$

Ex. Factor $64x^6 - 729$

$64x^6 = (4x^2)^3$

$729 = (9)^3$

$= (64x^2)^3 - (9)^3$

$a^3 - b^3$

$= (a-b)(a^2+ab+b^2)$

$= (4x^2-9)(16x^4+36x^2+81)$

$= (2x-3)(2x+3)(16x^4+36x^2+81)$

Assigned Work:

p.182 # 2adg, 3, 5, 7, 8