Rates of Change in Polynomial Functions

Recall:

(1) Average rate of change between $x = a$ and $x = b$ is
the slope of the secant line:

$$m_{\text{secant}} = \frac{f(b) - f(a)}{b - a = h}$$

$P_1(a, f(a))$

$P_2(b, f(b))$

(2) Instantaneous rate of change is the slope of the tangent
line at $x = a$, but can only be estimated:

$$m_{\text{tangent}} \doteq \frac{f(a+h) - f(a)}{h}$$

Improve the estimate by choosing smaller values of h.

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When estimating instantaneous rate of change, how do we
know if our estimate is "good enough"?

We need at least two estimates to compare against each
other. Ideally, there will be an obvious trend, which we can
use as our final estimate.

$$\%\text{diff} = \left| \frac{\text{accepted} - \text{estimate}}{\text{accepted}} \right| \times 100\%$$

Since we do not know the final, correct value, use your
best estimate as "accepted".

Ex.1 Estimate the iRoC at x = 4 to within 1%, for the polynomial
$$f(x) = 2(x - 3)(x - 2)(x + 5)$$

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① $h = 0.01$ $iRoC \doteq \dfrac{f(a+h) - f(a)}{h}$

   $a = 4$

   $a+h = 4.01$ $\doteq \dfrac{f(4.01) - f(4)}{0.01}$

                $\doteq \dfrac{36.582402 - 36}{0.01}$

                $\doteq 58.2402$

② $h = 0.001$ $iRoC \doteq \dfrac{f(4.001) - f(4)}{0.001}$

   $a = 4$

   $a+h = 4.001$ $\doteq \dfrac{36.058024 - 36}{0.001}$

                $\doteq 58.024002$

$\%.diff = \left| \dfrac{58.024002 - 58.2402}{58.024002} \right| \times 100\%$

      $= 0.37\%$ reasonable to assume within 1% of correct answer

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Some important rates of change
- rate of change of distance is speed
- rate of change of speed is acceleration

At a turning point (maximum or minimum), the instantaneous rate of change (slope on graph) will be zero.

Assigned Work:

p.235 # 5cde, 6cde, 7, 9, 11

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