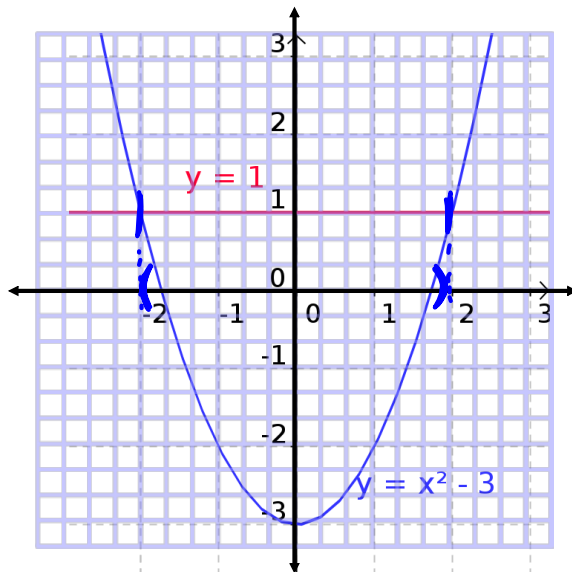


Solving Nonlinear Polynomial Inequalities

(1) Graphically: Graph both sides of the inequality and compare.

Ex.1 Solve $x^2 - 3 < 1$



Where is the graph of $y = x^2 - 3$ less than $y = 1$?

$-2 < x < 2$
 or
 $x \in (-2, 2) *$

Oct 5-1:44 PM

- (2) Algebraically:
 (a) Rearrange so one side is zero and solve the equation for both rational and irrational roots.
 (b) Use the zeroes to create intervals.
 (c) Test a value on each interval and see if it satisfies the original inequality.

Ex.2 Solve $x^2 - 3 < 1$ algebraically

(a) $x^2 - 4 < 0$ $x^2 - 4 = 0$
 $(x-2)(x+2) < 0$ $(x-2)(x+2) = 0$
 ① ② ③ $x = -2$ or $x = 2$
 $x < -2$ $-2 < x < 2$ $x > 2$

(b)

(c) test ①: try $x = -3$ $LS = (-3)^2 - 3$
 $LS < RS?$ $= 6$
 $6 < 1?$ FALSE $RS = 1$
 $x < -2$ → FAIL

test ②: try $x = 0$ $LS = 0^2 - 3$ $RS = 1$
 $LS < RS?$ $= -3$
 $-3 < 1?$ TRUE
 $-2 < x < 2$ → PASS

test ③: try $x = 4$ $LS = 4^2 - 3$ $RS = 1$
 $LS < RS?$ $= 13$
 $13 < 1?$ FALSE
 $x > 2$ → FAIL
 \therefore solution is $-2 < x < 2$
 or
 $x \in (-2, 2)$

Oct 5-1:54 PM

When examining intervals, we are only concerned with the sign, not the value, on each interval. It is often faster to use a factor table to test each interval.

Ex.3 Solve $x^3 - 2x^2 + 5x + 20 \geq 2x^2 + 14x - 16$
 (hint: factoring by grouping)

(a) $x^3 - 4x^2 - 9x + 36 \geq 0$

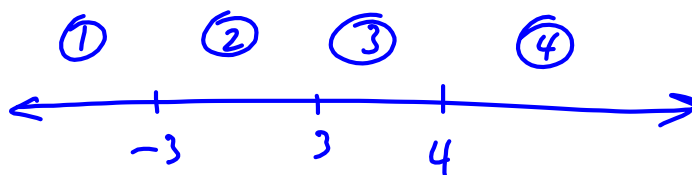
$x^2(x-4) - 9(x-4) \geq 0$

$(x-4)(x^2-9) \geq 0$

$(x-4)(x-3)(x+3) \geq 0$

consider $(x-4)(x-3)(x+3) = 0$

zeros: -3, 3, 4



Oct 5-2:12 PM

$x^3 - 2x^2 + 5x + 20 \geq 2x^2 + 14x - 16$
 $(x-4)(x-3)(x+3) \geq 0$

zeroes $x < -3$ $-3 < x < 3$ $x > 3$ $x > 4$

	test -4	test 0	test 3.5	test 5
$(x-4)$	-8	-	-	+
$(x-3)$	-7	-	+	+
$(x+3)$	-1	+	+	+
result	-56	+	-	+

FAIL PASS FAIL PASS

$x \in [-3, 3] \text{ or } [4, \infty)$

$x \in [-3, 3] \cup [4, \infty)$

union symbol \rightarrow "or"

$-3 < x < 3 \text{ or } x > 4$

Oct 5-2:18 PM

Assigned Work:

p.225 # [1bd, 2, 3] fundamentals

5, 6bdf, 7bdf (no graph or use tech), 8, 10, 13, 15

11 (use tech only), 18 [not taking up]

Sep 23-6:17 PM