Unit 3: Polynomial & Rational Equations & Inequalities

Solving Polynomial Equations

Recall: To solve an equation means finding the real roots of the equation.

When solving a quadratic equation, there are several options, such as:

- factoring to find the zeroes (roots)
- graphing
- completing the square (vertex form) and solving for y=0
- quadratic formula

Polynomial equations of degree 3 or higher can be solved by:

- graphing
- factoring down to degree 2 (quadratic), then applying one of the techniques listed above

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Unit 3: Polynomial & Rational Equations & Inequalities

Solving Polynomial Equations

(1) Rewrite the equation so it is equal to zero.
(2) Define the resulting polynomial as a function and apply the factor theorem.
(3) Factor out the first term (polynomial division), and repeat until in a fully factored form.
(4) Find the roots of the equation (i.e., set it back to zero and solve).
(5) Ignore solutions that are outside of the domain defined by the conditions of the problem.

Ex.1 Solve $3x^3 + 8x^2 = -3x + 2$

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Ex.1 Solve $3x^3 + 8x^2 = -3x + 2$

$3x^3 + 8x^2 + 3x - 2 = 0$ (f(x))

let $f(x) = 3x^3 + 8x^2 + 3x - 2$

roots? $\frac{\pm 1, 2}{1, 3}$

$\pm 1, 2, \frac{1}{3}, \frac{2}{3}$

$f(-1) = -3 + 8 - 3 - 2$

$= 0$

$x+1$ is a factor

$x+1 \overline{)\, 3x^3 + 8x^2 + 3x - 2}$

$3x^3 + 3x^2$ ↓

$5x^2 + 3x$

$5x^2 + 5x$

$-2x - 2$

$-2x - 2$

0

$f(x) = (x+1)(3x^2+5x-2)$

$= (x+1)(x+2)(3x-1)$

S 5
P -6
I 6, -1

$3x^2 + 6x - x - 2$

$= 3x(x+2) - 1(x+2)$

$= (x+2)(3x-1)$

$(x+1)(x+2)(3x-1) = 0$

$x = -1, \; x = -2, \; 3x - 1 = 0$

$3x = 1$

$x = \frac{1}{3}$

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Ex.2 Determine the exact roots of $x^3 - 4x^2 + 2x + 3$ (g(x))

roots? $\frac{\pm 1, 3}{1}$

$\pm 1, 3$

$x - 3$ is a factor

$g(-1) = -1 - 4 - 2 + 3$

$= -4$

$g(1) = 1 - 4 + 2 + 3$

$= 2$

$g(3) = 27 - 36 + 6 + 3$

$= 0$

$x - 3 \overline{)\, x^3 - 4x^2 + 2x + 3}$

$x^2 - x - 1$

$x^3 - 3x^2$

$-x^2 + 2x$

$-x^2 + 3x$

$-x + 3$

$-x + 3$

0

S -1
P -1
I ✗

QF?

$D = b^2 - 4ac$

$= 1 - 4(1)(-1)$

$= 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{1 \pm \sqrt{5}}{2}$

∴ solutions are $3, \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

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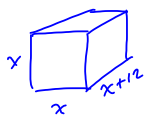
Ex.3 A box is in the shape of a rectangular prism. One side is a square, and the length is 12 units longer than the square sides. The volume of the box is 135 cubic units. What are the dimensions of the box?



$V = (x)(x)(x+12)$

$135 = x^2(x+12)$

$135 = x^3 + 12x^2$

$0 = x^3 + 12x^2 - 135$ $f(x)$

RR: $\pm 1, 3, 5, 9, 15, 27, 45, 135$ / 1

$f(3) = 27 + 108 - 135$ $(x-3)$

$= 0$

3	1	12	0	−135
	↓	3	45	135
	1	15	45	0

$x^2 + 15x + 45$

S 15
P 45
I ✗

$D = 225 - 180$

$= 45$

$x = \frac{-15 \pm \sqrt{45}}{2}$

possible solutions: $x = 3$, $x = \frac{-15+\sqrt{45}}{2}$, $x = \frac{-15-\sqrt{45}}{2}$

$x \doteq -4.146$ reject negative

$\therefore$ box is $3 \times 3 \times 15$ units.

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Assigned Work:

p.204 # 6, 7ad, 10, 13, 16

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