Graphs of Rational Functions

Use these key characteristics to sketch a rational function.
In general, use factored form to identify:
(1) discontinuities (holes)
(2) x-intercepts (zeroes) and y-intercept $x=0$
(3) asymptotes (vertical, horizontal, oblique)
(4) positive and negative intervals
(an interval table with zeroes & VAs can help with this)
(5) end behaviour, behaviour at any VAs

$h(x) = \frac{f(x)}{g(x)}$

$\frac{0}{0}$ $\frac{k}{0}$

VA: $x = 2$
try 2.1, 2.01
1.9, 1.99

Ex.1 Sketch the graph of $f(x) = \frac{4x^1 - 10}{2x^1 + 5}$

$= \frac{2(2x-5)}{2x+5}$

HA or OA: long division

$O_N = 1$
$O_D = 1$

$$\begin{array}{r|l} & 2 \\ 2x+5 & 4x-10 \\ & 4x+10 \\ \hline & -20 \end{array}$$

$\frac{4x-10}{2x+5} = 2 - \frac{20}{2x+5}$

HA: $y=2$

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Many rational functions are in the form: (linear divided by linear)

$$f(x) = \frac{ax+b}{cx+d}$$

Always start by looking for common factors and possible holes in graph.

Assuming no common factors between numerator and denominator, the equation of the horizontal asymptote will be:

$$\text{HA: } y = \frac{a}{c}$$

This can work with higher-order rational functions:
(note: order of numerator and denominator must be equal)

$$g(x) = \frac{3x^2 - 7x + 1}{2x^2 + 3x + 11} \qquad \text{HA: } y = \frac{3}{2}$$

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Ex.1 Sketch the graph of $f(x) = \dfrac{4x-10}{2x+5}$

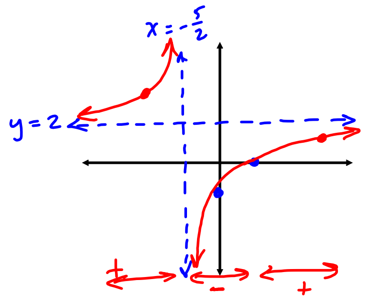
1. fully factor
2. ~~holes~~ & VAs
3. zeroes & y-int
4. behaviour

$$f(x) = \frac{2(2x-5)}{2x+5}$$

VA: $2x+5=0$

$2x = -5$

$x = \frac{-5}{2}$



HA: $y = 2$

zeroes: $2x-5=0$

$x = \frac{5}{2}$

y-int: set $x=0$

$f(0) = \frac{2(-5)}{5}$

$= -2$

	$(-\infty, -\frac{5}{2})$	$-\frac{5}{2}$ $(-\frac{5}{2}, \frac{5}{2})$	$\frac{5}{2}$ $(\frac{5}{2}, \infty)$
+2	+	+	+
$2x-5$	−	−	+
$2x+5$	−	+	+
result	+	−	+

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Assigned Work:

p.272 # 1, 3, 5bcd, 6bd, 14
+ worksheet

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