Quotients of Polynomial Functions
(Rational Functions)

Rational functions can be expressed as $f(x) = \dfrac{p(x)}{q(x)}$

where $p(x)$ and $q(x)$ are polynomial functions.

With the function q(x) in the denominator, we need to consider any discontinuities where q(x) = 0.

(a) A hole will occur at $x = a$ if both p(x) and q(x) have a common factor of $(x - a)$.

(b) A vertical asymptote will occur at $x = a$ when $\dfrac{p(a)}{q(a)} = \dfrac{k}{0}$ (i.e., a constant over zero).

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There are also horizontal and oblique asymptotes, which do not affect continuity. Instead, they determine the end behaviour of the rational function.

as $x \to \infty$
as $x \to -\infty$

(3) The function $f(x) = \dfrac{p(x)}{q(x)}$ has a horizontal asymptote (HA)

if $\text{order of } p(x) \leq \text{order of } q(x)$

To determine the equation, divide the numerator by the denominator (long division, synthetic). Consider end behaviour (generally means we discard the remainder).

Ex.1 Determine the HA for $f(x) = \dfrac{2x}{x+1}$

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Ex.1 Determine the HA for $f(x) = \dfrac{2x}{x+1}$

Steps:
~~(1)~~ Perform Division
~~(2)~~ Express as Quotient & Remainder
(3) End Behaviour

$$\begin{array}{r} 2 \\ x+1 \overline{)\, 2x + 0} \\ \underline{2x + 2} \\ -2 \rightarrow R \end{array}$$

$$\frac{P(x)}{D} = Q + \frac{R}{D}$$

$$f(x) = 2 + \left(\frac{-2}{x+1}\right)$$

$$= 2 - \frac{2}{x+1} \rightarrow 0$$

as $x \to \infty$, $f(x) \to 2$

as $x \to -\infty$, $f(x) \to 2$

HA: $y = 2$

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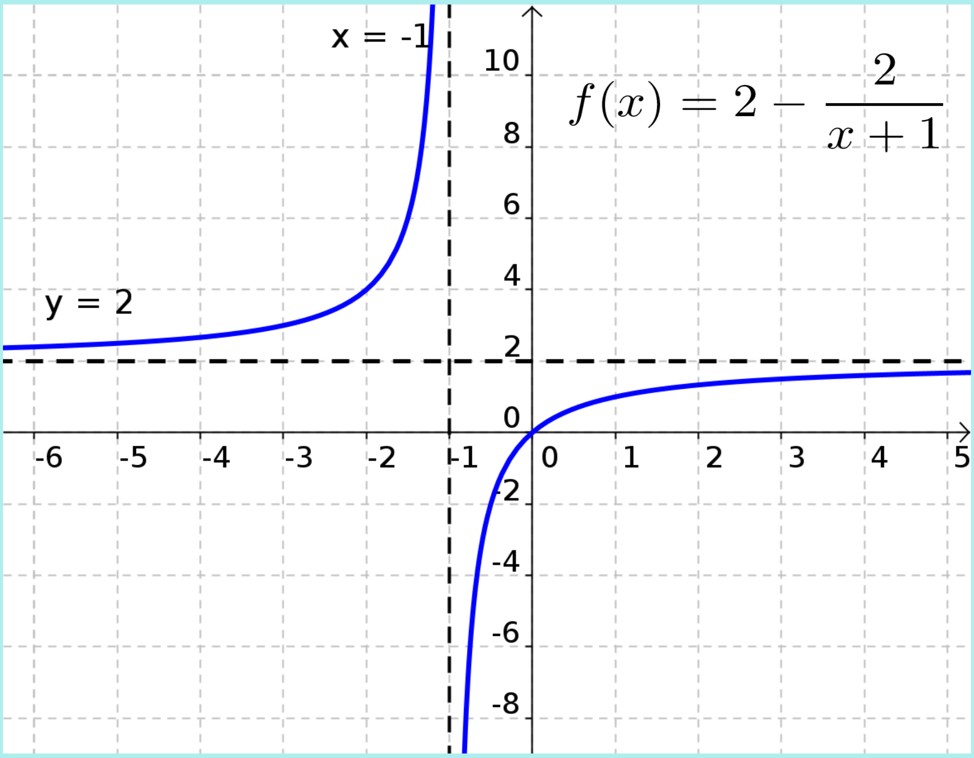
Ex.1 Determine the HA for $f(x) = \dfrac{2x}{x+1}$



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(4) An oblique asymptote (OA) will occur only if

$$\text{degree of } p(x) > \text{degree of } q(x) \text{ by exactly } 1$$

Ex.2 Determine the OA for $f(x) = \dfrac{x^2+4}{x+1}$

Steps:
(1) Perform Division
(2) Express as Quotient & Remainder
(3) End Behaviour

$$\begin{array}{r} x - 1 \\ x+1 \overline{) \, x^2 + 0x + 4} \\ \underline{x^2 + x} \quad \downarrow \\ -x + 4 \\ \underline{-x - 1} \\ 5 \rightarrow R \end{array}$$

$$f(x) = x - 1 + \frac{5}{x+1} \rightarrow 0$$

as $x \to \infty$, $(x+1) \to \infty$

$$\frac{5}{\infty} \to 0^+$$

as $x \to -\infty$, $(x+1) \to -\infty$

$$\frac{5}{-\infty} \to 0^-$$

OA: $y = x - 1$

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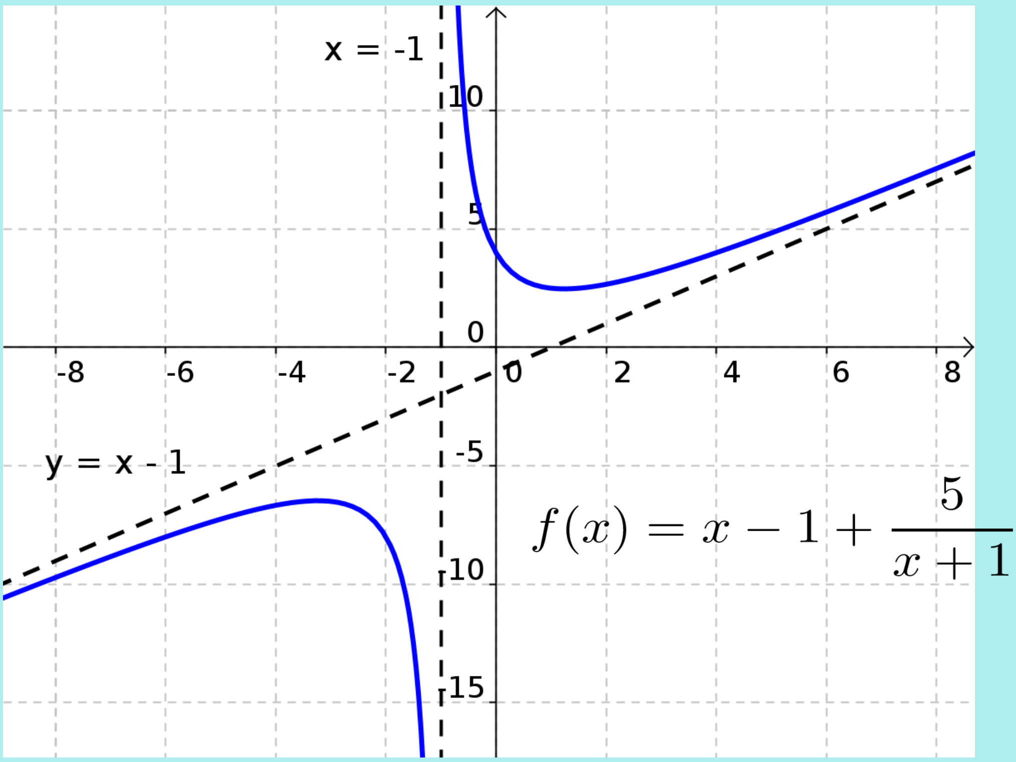
Ex.2 Determine the OA for $f(x) = \dfrac{x^2+4}{x+1}$



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Assigned Work:

p.262 # 1, 2, 3

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