Solving Exponential and Logarithmic Equations Dec. 7/2016

The definition and properties of logarithms can be used to solve equations in which either powers or logarithms appear. If the unknown occurs in an exponent then the strategy is to isolate it by taking the logarithm of both sides.

Ex.1 Solve $3^{x+2} = 4$
(a) using definition of logarithms.
(b) by taking the log (base 10) of both sides.

check your solution!

(a) $3^{x+2} = 4$ ($a = 3$, $n = x+2$, $m = 4$)

$\log_a m = n$
$a^n = m$

$x+2 = \log_3 4$

$x = (\log_3 4) - 2$

$x = \frac{(\log 4)}{(\log 3)} - 2$ exact

$x \doteq -0.738$ approx.

(b) $3^{x+2} = 4$

$\log(3^{x+2}) = \log 4$

$(x+2)\log 3 = \log 4$

$x+2 = \frac{\log 4}{\log 3}$

$x = \frac{\log 4}{\log 3} - 2$

Ex.2 Solve $\log_2 x - \log_2 3 = \log_2 6$

$\log_2\left(\frac{x}{3}\right) = \log_2 6$

$\Rightarrow \frac{x}{3} = 6$

$x = 18$

$\log_2 x = \log_2 6 + \log_2 3$

$\log_2 x = \log_2(6 \cdot 3)$

$\log_2 x = \log_2 18$

$\Rightarrow x = 18$

$2^{\log_2 x} = 2^{\log_2 18}$

$x = 18$

Ex.3 Solve $6^{3x} = 4^{2x-3}$

$\log(6^{3x}) = \log(4^{2x-3})$

$(3x)\log 6 = (2x-3)\log 4$

$3x\log 6 = 2x\log 4 - 3\log 4$

$3x\log 6 - 2x\log 4 = -3\log 4$

$\frac{x(3\log 6 - 2\log 4)}{(3\log 6 - 2\log 4} = \frac{-3\log 4}{3\log 6 - 2\log 4}$

$x = \frac{(-3\log 4)}{(3\log 6 - 2\log 4)}$

$x \doteq -1.5979$

Ex.4 Solve $\log_x 0.04 = -2$

a m n

$m = a^n$

$n = \log_a m$

$0.04 = x^{-2}$

$0.04 = \frac{1}{x^2}$

$x^2 = \frac{1}{0.04}$

$x^2 = 25$

$x = \pm 5$

$x = 5$ (x is base of $\log_x$)

Ex.5 Solve $\log(x+2)+\log(x-1)=1$

check your solution!

$\log_{10}[(x+2)(x-1)] = 1$

Assigned Work:

p.485 # 2, 8, 10 b c, 17
p.491 # 4, 5, 7 d, 12

for next class

p.485 # 4, 6bc, 7, 11
p.492 # 3, 9 b

work period?

p.485 - 10b

$x = \log_3 25$

$= \dfrac{\log_{10} 25}{\log_{10} 3}$

485 - 17c

$$3(2)^x = 4^{x+1}$$

$$3(2)^x = (2^2)^{x+1}$$

$$\frac{3(2)^x}{2^x} = \frac{2^{2x+2}}{2^x}$$

$$3 = 2^{x+2}$$

$$x+2 = \log_2 3$$

$$x+2 = \frac{\log 3}{\log 2}$$

$$x = \frac{\log 3}{\log 2} - 2$$

485 - 17c

$$3(2)^x = 4^{x+1}$$

$$\log\left[3(2)^x\right] = \log 4^{x+1}$$

$$\log 3 + \log 2^x = (x+1)\log 4$$

$$\log 3 + x\log 2 = x\log 4 + \log 4$$

$$\vdots$$

$$x = \underline{\qquad\qquad}$$

491 - 7d

$$\log(2x+1) + \log(x-1) = \log 9$$

$$\log\left[\underbrace{(2x+1)(x-1)}\right] = \log \underbrace{9}$$

$$\Rightarrow (2x+1)(x-1) = 9$$

$$\vdots$$

$$= 0$$

$$\vdots$$

$$(\quad)(\quad) = 0$$

p.485 #4

$$m(t) = P\left(\tfrac{1}{2}\right)^{\frac{t}{h}}$$

$h = 8$ hours

$P = 300g$

(a) $m(t) = 200$

$$200 = 300\left(\tfrac{1}{2}\right)^{\frac{t}{8}}$$

$y = a^x$

$$\frac{2}{3} = \left(\tfrac{1}{2}\right)^{t/8}$$

$$\log \frac{2}{3} = \log\left(\tfrac{1}{2}\right)^{t/8}$$

$$\log \frac{2}{3} = \frac{t}{8} \log \frac{1}{2}$$

$$\frac{\log \frac{2}{3}}{\log \frac{1}{2}} = \frac{t}{8}$$

$$t = \frac{8 \log \frac{2}{3}}{\log \frac{1}{2}}$$

$t \doteq$ ________

485 6c

$A = P(1+i)^n$

principal, interest rate PER compound period, # of compound periods

$i = \frac{10\%}{4}$

$= 2.5\%$

$= 0.025$

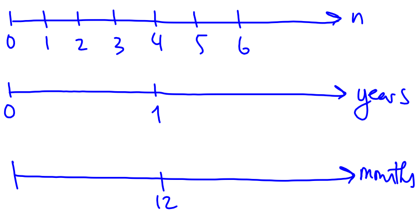
$7500 = 5000(1+0.025)^n$ $\quad y = a^x$

$\frac{7500}{5000} = 1.025^n$

$\log \frac{3}{2} = n \log 1.025$

$n = \frac{\log \frac{3}{2}}{\log 1.025}$

$n \doteq$ ____

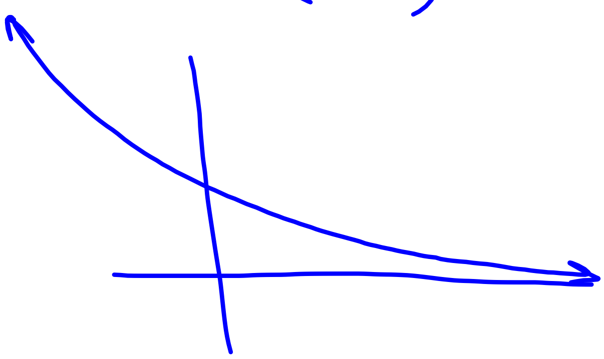


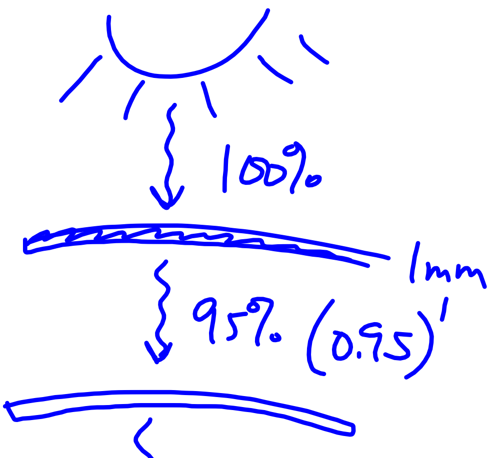
p.485 #11

5% for 1mm

$(1-0.05)^x$

$I = I_0(0.95)^x$





95% of 95%

$\rightarrow (0.95)(0.95)$

$\rightarrow (0.95)^2$

p.492 #9(b)

$$L = 10\log\left(\frac{I}{I_0}\right)$$

$$I_0 = 10^{-12}\ w/m^2$$

(b)

$$84 = 10\log\frac{I}{I_0}$$

$$y = \log_a x$$

$$8.4 = \log\frac{I}{I_0}$$

$$\frac{I}{I_0} = 10^{8.4}$$

$$I = I_0 \times 10^{8.4}$$

$$I = (10^{-12})(10^{8.4})$$

$$I = 10^{8.4-12}$$

$$I = 10^{-3.6}$$

$\therefore$ the music is $10^{-3.6}\ w/m^2$