Laws of Logarithms

Recall: Exponent Laws

same base

$$(a^x)(a^y) = a^{x+y}$$

$$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}, \quad a \neq 0$$

$$a^{-x} = \frac{1}{a^x}, \quad a \neq 0$$

$$\left(a^x\right)^y = a^{xy}$$

$$a^0 = 1, \quad a \neq 0$$

different bases

$$(ab)^x = \left(a^x\right)\left(b^x\right)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, \quad b \neq 0$$

Ex. Derive the Product Law for Logarithms (use the product law for exponents)

$$(a^x)(a^y) = a^{x+y}$$

let $m = a^x$ $\quad n = a^y$ $\quad y = a^x$

* $x = \log_a m$ * $y = \log_a n$ $\quad x = \log_a y$

$a^x a^y = a^{x+y}$

$m\,n = a^{x+y}$

* $x + y = \log_a (mn)$

$$\boxed{\underbrace{\log_a m + \log_a n}_{\text{sum of logarithms}} = \log_a mn}$$

log of a product

Ex. Derive the Quotient Law for Logarithms $\frac{a^x}{a^y} = a^{x-y}$
(use the quotient law for exponents)

$m = a^x$ $n = a^y$

$x = \log_a m$ $y = \log_a n$

$y = a^x$

$x = \log_a y$

$\frac{a^x}{a^y} = a^{x-y}$

$\frac{m}{n} = a^{x-y}$

$x - y = \log_a\left(\frac{m}{n}\right)$

$\boxed{\underbrace{\log_a m - \log_a n}_{\text{difference of logs.}} = \underbrace{\log\left(\frac{m}{n}\right)}_{\text{log of a quotient}}}$

Ex. Derive the Power Law for Logarithms $\left(a^x\right)^y = a^{xy}$
(use the power law for exponents)

let $m = a^x$

$x = \log_a m$

$(a^x)^y = a^{xy}$

$(m)^y = a^{xy}$

$xy = \log_a(m^y)$

$(\log_a m)y = \log_a(m^y)$ ✓ ☺

$\boxed{y \log_a m^y = \log_a m^y}$

Laws of Logarithms

product law: $\log_a xy = \log_a x + \log_a y$

quotient law: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

power law: $\log_a x^r = r\log_a x$

Ex.1 Simplify then evaluate (no log calculations!):

(a) $\log_3 6 + \log_3 4.5$

(b) $\log_2 48 - \log_2 3$

(c) $\log_5 \sqrt[3]{25}$

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$\log_a xy = \log_a x + \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^r = r\log_a x$

(a) $\log_3 6 + \log_3 4.5$

$= \log_3 (6(4.5))$

$= \log_3 27$

$= \log_3 3^3$

$= 3$

$\log_a a^x = x$

(b) $\log_2 48 - \log_2 3$

$= \log_2\left(\frac{48}{3}\right)$

$= \log_2 16$

$= \log_2 2^4$

$= 4$

(c) $\log_5 \sqrt[3]{25}$

$= \log_5 25^{\frac{1}{3}}$

$= \frac{1}{3}\log_5 25$

$= \frac{1}{3}\log_5 5^2$

$= \frac{1}{3}(2)$

$= \frac{2}{3}$

Ex.2 Use the power law to show $\underbrace{\log_a x} = \dfrac{\log_{10} x}{\log_{10} a}$ ✓

$$\boxed{\log_a x^r = r\log_a x}$$

$y = \log_a x$

$x = a^y$ take $\log_{10}$ of both sides

$\log_{10} x = \log_{10} a^{y}$

$\log_{10} x = y \log_{10} a$

$\dfrac{\log_{10} x}{\log_{10} a} = y$

$\dfrac{\log_{10} x}{\log_{10} a} = \log_a x$

Ex.3 Rewrite as a single log to a common base:

$$\log 12 + \frac{1}{2}\log 7 - \log 2$$

$= \log_{10} 12 + \log_{10} 7^{\frac{1}{2}} - \log_{10} 2$ ① ②

$= \log\left[12(\sqrt{7})\right] - \log 2$

$= \log\left[\dfrac{\cancel{12}^{6}\sqrt{7}}{\cancel{2}_{1}}\right]$

$= \log\left[6\sqrt{7}\right]$

$= \log\left(6\sqrt{7}\right)$

$\log_a xy = \log_a x + \log_a y$

$\log_a \left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^r = r\log_a x$

Order of Operations

BEDMAS

PEDMAS

PEMDAS

same "level"

evaluate left to right

Assigned Work:

p.475 # 5 (look past obvious answer!),
6, 7, 9ace, 10ace, 11bde, 17