Evaluating Logarithms

$y = \log_a x$ is equivalent to $x = a^y$

For many problems, we can obtain an exact value by switching between these equivalent expressions and looking for a common base.

There are also some general rules we can develop.

Ex.1 Solve

(a) $y = log_3 3^2$

y a x

$x = a^y$

$3^2 = 3^y$

$\Rightarrow y = 2$

(b) $y = log_4 4^7$

y a x

$x = a^y$

$4^7 = 4^y$

$\Rightarrow y = 7$

$y = \log_a x$ is equivalent to $x = a^y$

In general: $log_a a^x = x$ (1)

Ex.2 Evaluate:

(a) $log_{10} 1$

$1 = 10^0$

$= \log_{10} 10^0$

$= 0$

(b) $log_5 1$

$1 = 5^0$

$= \log_5 5^0$

$= 0$

$\log_5 1 = y$

a x y

$1 = 5^y$

$5^0 = 5^y \Rightarrow y = 0$

$y = \log_a x$ is equivalent to $x = a^y$

In general: $\log_a 1 = 0$ (2)

$a^0 = 1$

Ex.3 Evaluate:

(a) $2^{\log_2 x}$ $= x$

(b) $5^{\log_5 x}$ $= x$

$7^{\log_7 13} = 13$

$\log_5 x = \log_5 y$

$\Rightarrow x = y$

2^x $\log_2 x$

inverses undo each other..

$x = \log_a y$ $y = a^x$

$y = \log_a x$ is equivalent to $x = a^y$

In general: $a^{\log_a x} = x$ (3)

What if no common base is possible, and these general rules cannot be applied?

Recall: Many calculators only allow for a base of 10 or 'e'.

$y = \log_{10} x$ or $y = \ln x$

$\log x$ $\log_e x$

For different bases, we can still calculate the value of a logarithm by using an equivalent expression.

$$\log_a x = \frac{\log_{10} x}{\log_{10} a}$$ (4)

Note: we will derive this in our lesson on "laws of logarithms"

Ex.4 $\log_7 15 = \dfrac{\log_{10} 15}{\log_{10} 7}$

$\doteq \dfrac{1.17609}{0.84510}$

$\doteq 1.39$

check: $7^{1.39} \doteq 14.95$ ✓

Assigned Work:

p.466 # 3 (using rule 1 or 2)
5 (using 1 or 2)
8 (using 4)
6, 9, 12, 17