WS – Logarithmic & Exponential Functions   Dec 13, 2017

1. You win $1.5 million in a lottery and invest it in a GIC that earns 5% per annum, compounded yearly.
   (a) What is it worth after 4 years? ($1,823,259.38)
   (b) When will it double in value? (14.2 years)

2. Your new car cost $23,000 new and depreciates (i.e., loses value) 30% annually. You want to trade it in when it reaches a value of $10,000. How long should you keep the car? (2.3 years)

3. A weight-loss program claims that, after the first two days, weight loss is exponential. The weight loss is expressed as a percent of the client's changing weight. A client who is participating in the program has a weight of 98 kg four days after starting the program. After 20 days, the weight is 95 kg.
   (a) What will the weight be after 40 days? (91.2kg)
   (b) What is the rate of weight loss? Express this value as a percent per day. (about 0.2% per day)

4. Newton's Law of Cooling: $T=(T_0-S)e^{kt}+S$, where T is the final temperature, T_0 is the initial temperature, S is the temperature of the surroundings, k is a constant, and t is time.

   Suppose you are sitting outside on a summer day reading a book. The book reaches a temperature of 30°C. When you bring the book indoors (22°C), it takes 20 minutes before the book cools to 25°C. How much more time will pass before the book reaches 22.5°C? (36.6 minutes)

5. The logistic equation, $P(t)=\dfrac{A}{1+be^{-kt}}$, is a function that models a population in which the growth is initially rapid but eventually levels off. The growth of a bacterial culture is modelled where A = 6, b = 6, and k = 0.3, where P(t) models the mass of the culture in grams.
   (a) What is the initial mass of the culture? (0.857 g)
   (b) Graph P(t) using graphing technology, and use your graph to show that the maximum mass is 6 grams.

6. Karen packs her lunch each morning at 7am and includes a frozen juice box (-10°C). At school, the lunch sits in her locker at room temperature (20°C) until lunch (noon). At 10am, the juice is still frozen (-2°C). What will the juice temperature be at lunch? (2.1°C)

7. The following data represents a car's velocity, $v(t)$, after the driver steps on the brake, where t measures how long the driver has been braking in seconds.

Time (s)	2	4	6	8	10
Velocity (km/h)	31	20	14	9	5

   (a) Create an algebraic model for this data. $v(t)=(49.8675)(0.8006)^t$
   (b) Determine when the driver has 'stopped' the car. The model fails for v=0, so solve for v=0.1. (t = 28 s)
   (c) What was the initial velocity when the brakes were first applied? (50 km/h)

8. Kenneth paid $16,000 for a car which depreciates 30% per year. At the same time, Karen paid $8,000 for a painting which will appreciate 3% per year. When will their investments be worth the same amount? (1.79 years)

9. The spread of a flu virus through a population of 1000 people is modelled by $N(t)=\dfrac{1000}{1+990\,e^{-0.7t}}$, where N(t) is the total number infected after t days. How long will it take for 901 people to be infected? (13 days)