

Quotients of Functions

$$(f \div g)(x) = f(x) \div g(x)$$

Ex.1 Given $f = \{(1, 5), (2, 0), (4, -5), (5, 11)\}$

$$g = \{(2, -4), (3, 1), (5, 0)\}$$

$y = 0$ in denominator

(a) determine the domain of f , g , and $(f \div g)$

(b) determine $(f \div g)$

$$(a) D_f = \{1, 2, 4, 5\} \quad D_g = \{2, 3, 5\}$$

$$D_{f \div g} = \{2\}$$

$\frac{y_f}{y_g} = \frac{0}{-4} = 0 \checkmark$

$$(b) f \div g = \{(2, 0)\}$$

$\frac{y_f}{y_g} = \frac{11}{0}$
undef.

Note: Since $g(x)$ is in the denominator, and we must avoid division by zero, the domain has an additional condition:

$$D_{f \div g} = D_f \cap D_g, g(x) \neq 0$$

y -coordinate in g

$$\text{Ex.2 Given } f(x) = x^2 + x - 6 \quad D_f = \{x \in \mathbb{R}\}$$

$$g(x) = \sqrt{x+3} \quad D_g = \{x \in \mathbb{R} | x \geq -3\}$$

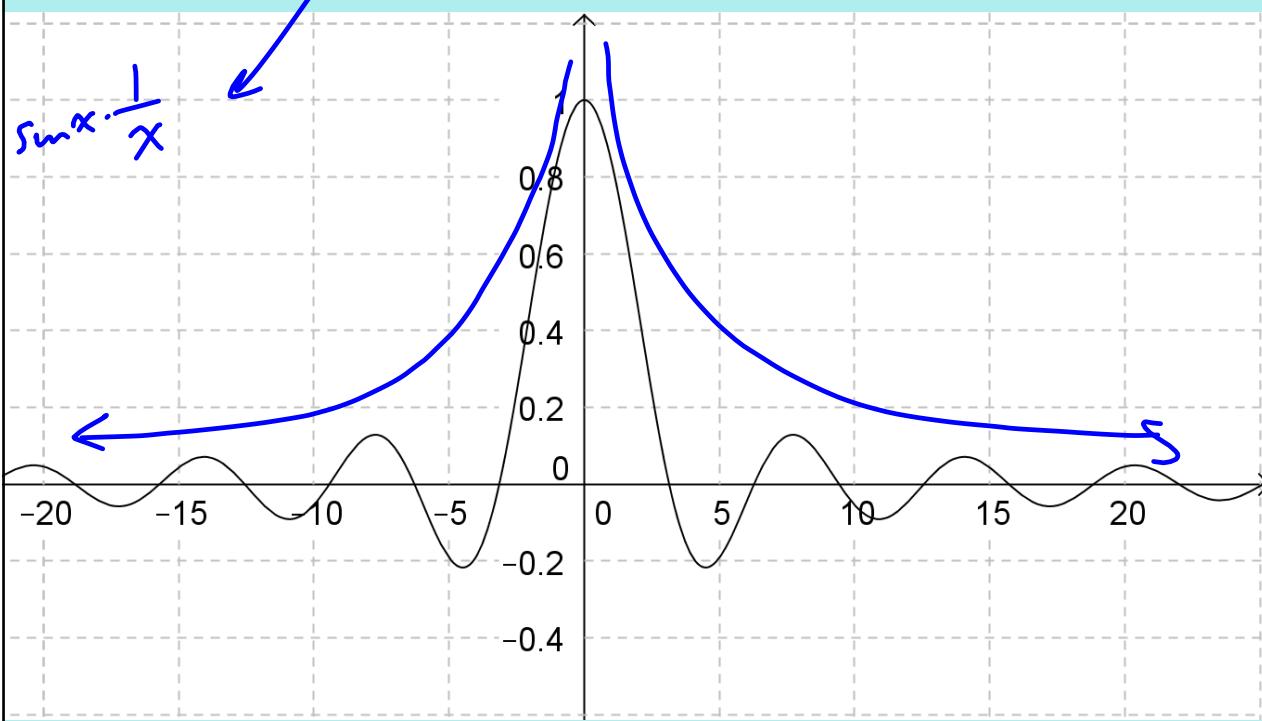
determine $(f \div g)(x)$ and state the domain.

$$\begin{aligned}
 (f \div g)(x) &= \frac{f(x)}{g(x)} \\
 &= \frac{x^2 + x - 6}{\sqrt{x+3}} \quad \sqrt{x+3} \neq 0, \quad x \neq -3 \\
 &= \frac{(x+3)(x-2)}{\sqrt{x+3}} \\
 &= \frac{(x+3)^{\frac{1}{2}}(x-2)}{(x+3)^{\frac{1}{2}}} \\
 &= (x+3)^{\frac{1}{2}}(x-2) \quad \text{created a hole at } x = -3 \\
 &= \sqrt{x+3}(x-2)
 \end{aligned}$$

$$D_{f \div g} = \{x \in \mathbb{R} | x > -3\}$$

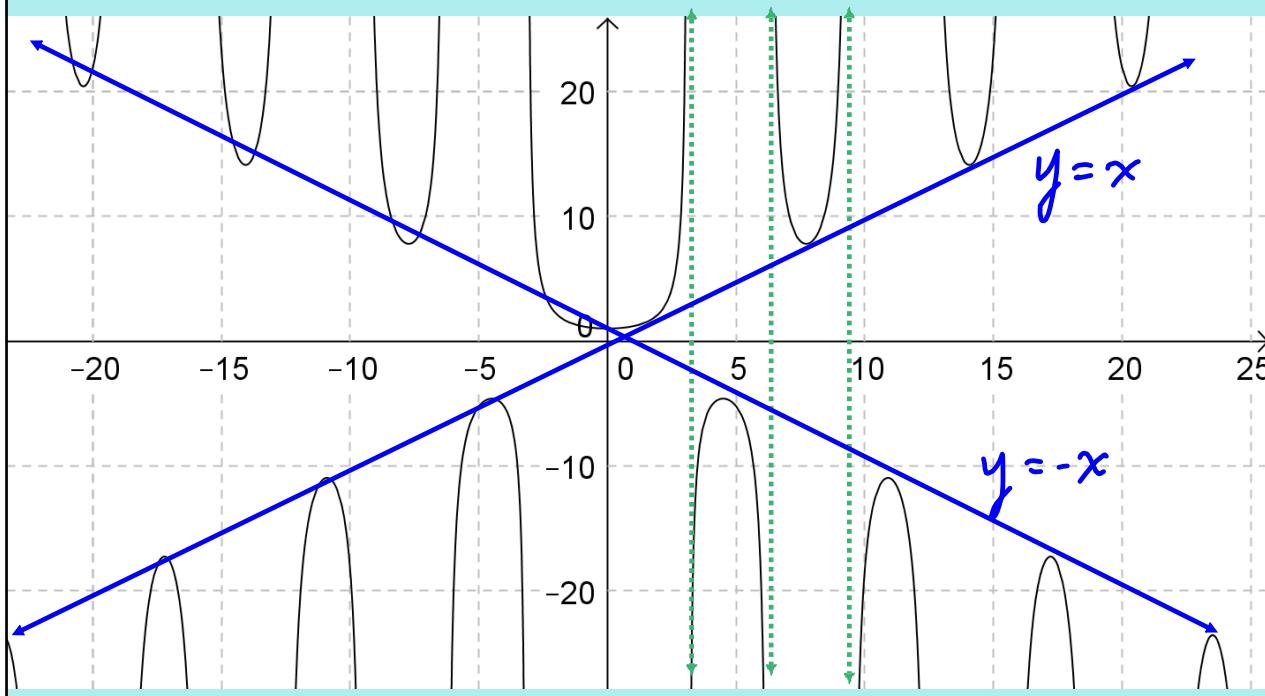
Example: $\frac{\sin x}{x}$

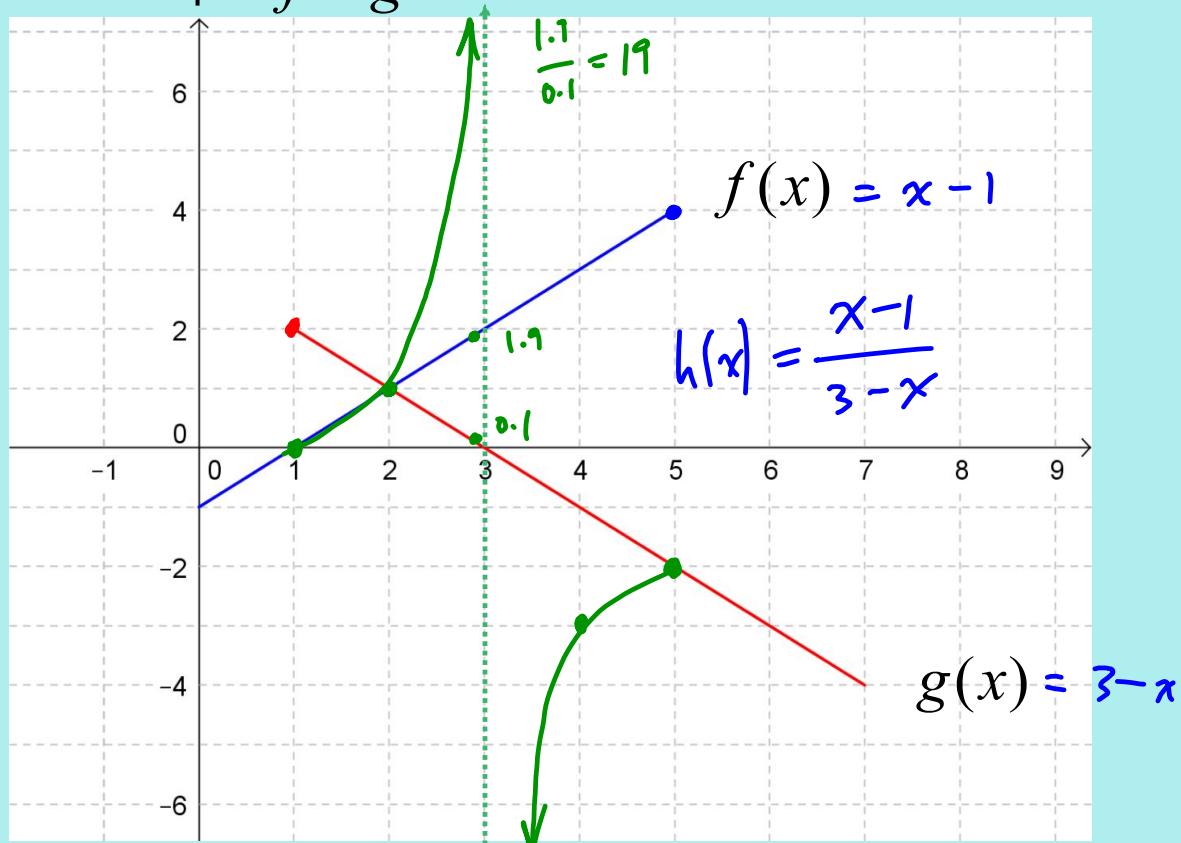
$$D = \{x \in \mathbb{R} \mid x \neq 0\}$$



Example: $\frac{x}{\sin x}$

$$D = \{x \in \mathbb{R} \mid x \neq n\pi, n \in \mathbb{Z}\}$$



Ex.3 Graph $f \div g$ 

Assigned Work:

p.542 # 1, 2, 3

p.544 # 3, 4, 6