Products of Functions

$$(f \times g)(x) = f(x) \times g(x)$$

"f times g of x"

The domain of the combined function is the intersection of the domains of the original functions.

$$D_{f \times g} = D_f \cap D_g$$

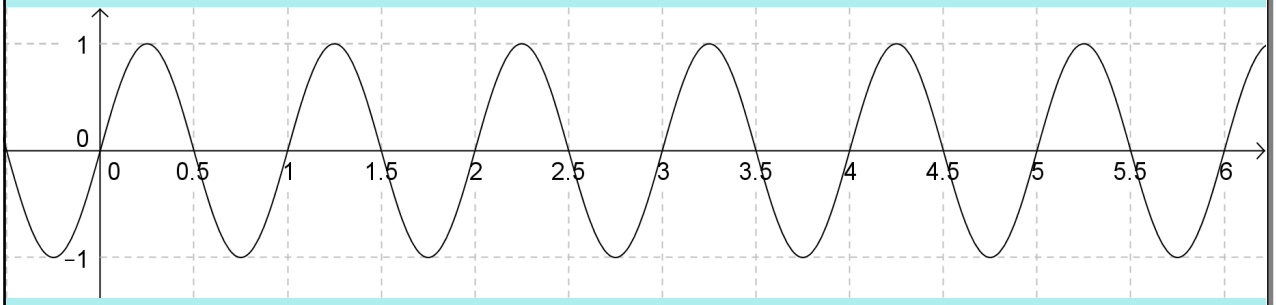
General Techniques:

(1) Graphing: Multiply y-values for common x-values.

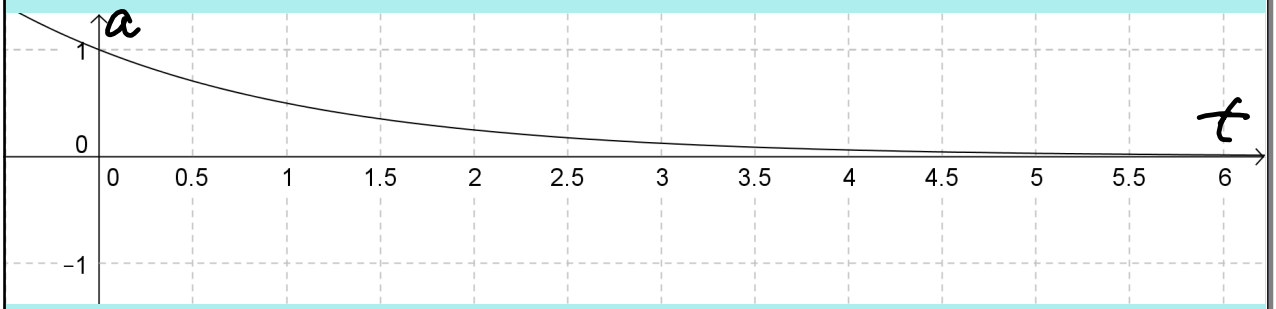
(2) Algebra: Multiply expressions, simplify product, state restrictions (i.e., domain) based on intersection of original domains.

Application - Damped Harmonic Motion

A perfect spring will oscillate forever:



More realistically, the oscillation will decay over time:



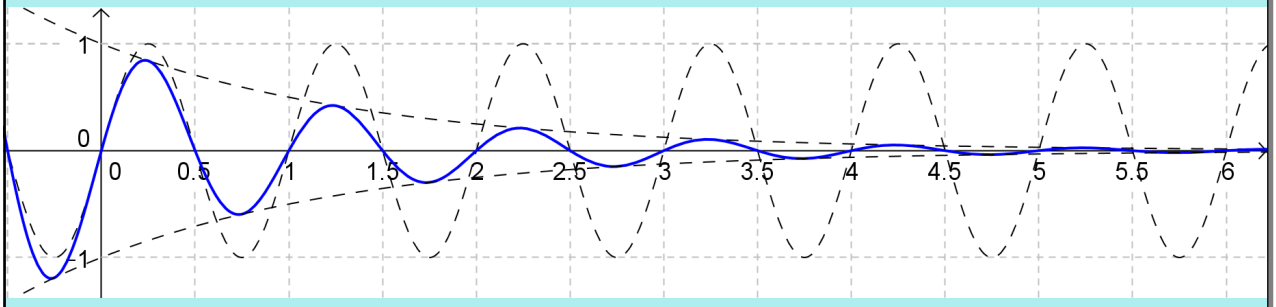
Application - Damped Harmonic Motion

Periodic Function: $f(x) = \sin(2\pi x)$

Exponential Decay: $g(x) = 2^{-x} = \left(\frac{1}{2}\right)^x$

Damped Harmonic Function:

$$(f \times g)(x) = 2^{-x}\sin(2\pi x)$$



Ex.1 Given $f = \{(1,3),(2,-5),(3,7)\}$

$g = \{(2,-2),(3,3),(4,1)\}$

note: same general idea as graphing

determine $f \times g$.

$D_f = \{1, 2, 3\}$

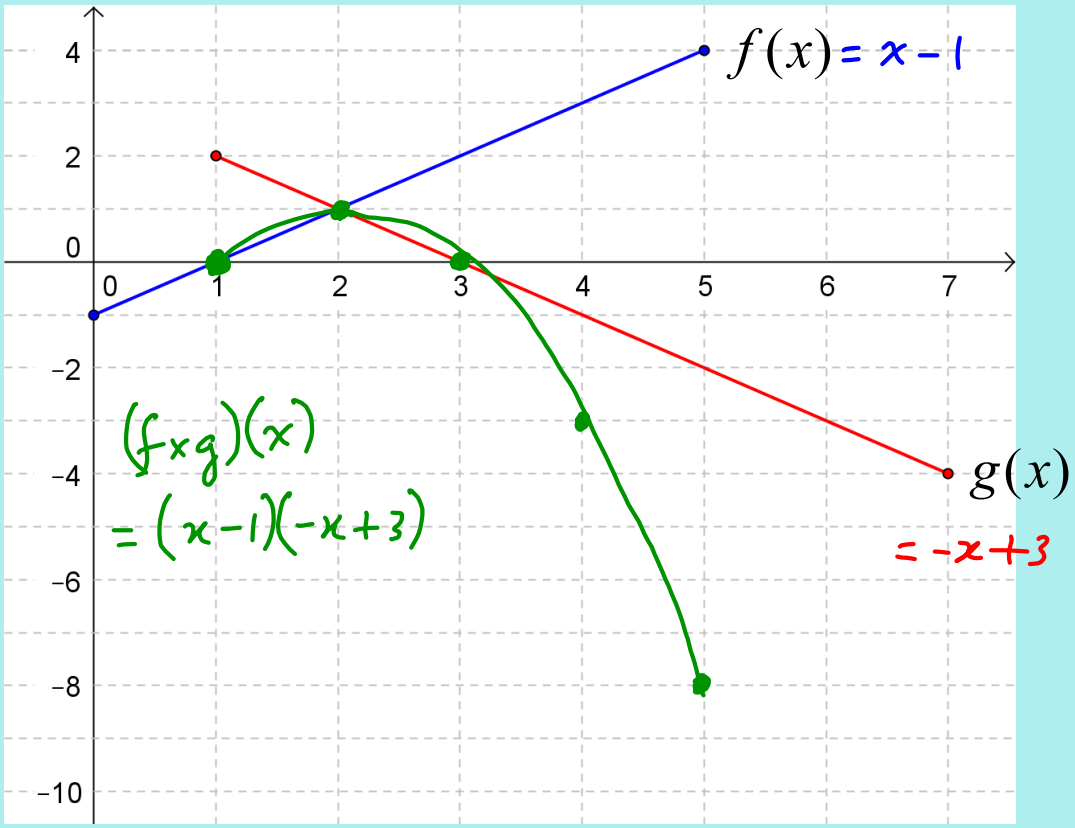
$D_g = \{2, 3, 4\}$

$D_{f \times g} = D_f \cap D_g$

$= \{2, 3\}$

$f \times g = \{(2, 10), (3, 21)\}$

Graph $f \times g$



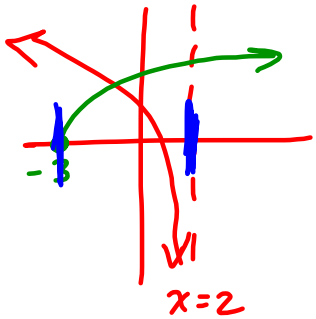
Ex.2 Given $f(x) = \log(2 - x)$ = log [-(x - 2)]
$g(x) = \sqrt{x + 3}$

h. reflect, right 2

determine $(f \times g)(x)$ and state the domain.

$(f \times g)(x) = [\log(2-x)][\sqrt{x+3}]$

$= \sqrt{x+3}\,\log(2-x)$



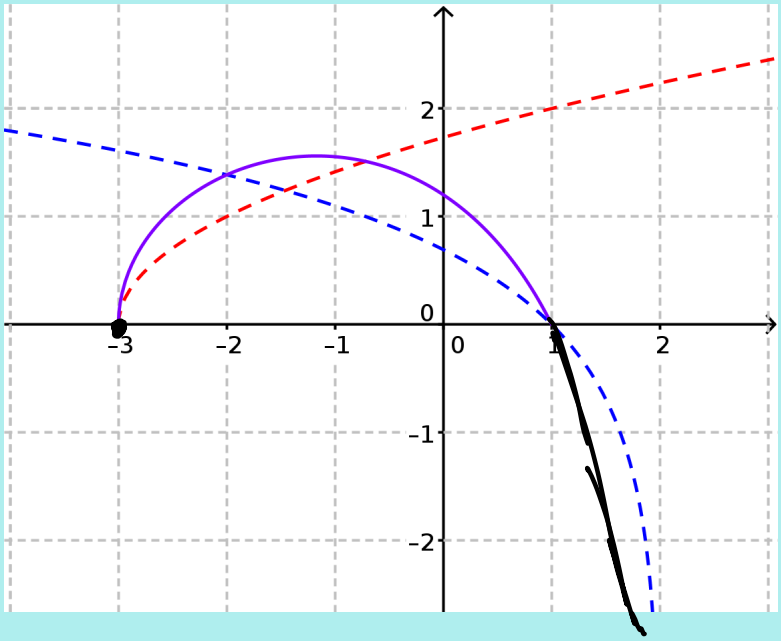
$D_f = \{x \in \mathbb{R} \mid x < 2\}$

$D_g = \{x \in \mathbb{R} \mid x \geq -3\}$

$D_{f \times g} = \{x \in \mathbb{R} \mid -3 \leq x < 2\}$

Ex.2 Given $f(x) = \log(2 - x)$
$g(x) = \sqrt{x + 3}$

determine $(f \times g)(x)$ and state the domain.



Assigned Work:

p.537 # 1ae, 3, 4bef, 5bef, 6ef, 8ad, 11, 12