MHF 4U, Grade 12 Advance Functions
Exam Review

Part A: Polynomial & Rational Functions

1. Determine the equation of the family of polynomial functions with zeros -2, 0, and 3.
2. **True or False:** -3 is a root of the polynomial function $f(x) = x^4 - 5x^2 + 12x + 16$.
3. **True or False:** $x + 2$ is a factor of the polynomial $x^3 - 5x^2 - 22x - 16$.
4. Determine the vertical asymptote of the reciprocal of $f(x) = 3x + 2$.
5. Determine the equation of the horizontal asymptote of $f(x) = \dfrac{4x - 3}{2x + 5}$.
6. **True or False**: $f(x) = \dfrac{x^2 - 9}{x - 3}$ has an oblique asymptote.
7. Use the factor theorem and long division to fully factor $x^3 + 9x^2 + 8x - 60$.
8. Determine an expression for $f(x)$ in which $f(x)$ is a quartic function, $f(x) > 0$ when $-3 < x < -1$, $f(x) \le 0$ when $x < -3$ or $x > -1$, $f(x)$ has a double root when $x = 5$, and $f(4) = -70$.
9. The bacteria count, g, in thousands, for a sample of pond water, that is left to sit over time, t, in days can be modeled by the equation $g(t) = \dfrac{15t}{t^2 + 9}, t > 0$. When will the bacteria count be greater than one thousand? Round your answer to one decimal place.

Part B: Trigonometric Functions

1. Convert 115° to radians. Express your answer as a fraction of π, in lowest terms.
2. State the exact value of $\tan \dfrac{7\pi}{6}$.
3. Determine the value of sec 2.37. Round your answer to 3 decimal places.
4. Determine the two exact radian values of θ, $0 \le \theta \le 2\pi$, for $\sin\theta = -\dfrac{\sqrt{3}}{2}$.
5. State the period, amplitude, vertical displacement and phase shift of the function $y = -2\sin\dfrac{2}{3}\left(x - \dfrac{\pi}{6}\right) - 5$.
6. Determine the exact solutions of $2\cos^2 x + \cos x - 1 = 0$ for x in the interval $0 \le x \le 2\pi$.
7. The population of blackflies at a lake in northern Ontario can be modeled by the function $P(t) = 23.7\cos\left(\dfrac{\pi}{6}(t - 7)\right) + 24.1$, where P is in millions and t is in months. When is the population more than 40 million? Round answers to the nearest day.
8. Solve $\sin 2x + 2\cos^2 x = 0$ for $0 \le x \le 2\pi$.

Part C: Exponential & Logarithmic Functions

1. Evaluate $\log 23$. Round your answer to three decimal places.
2. Express $\log_5 15 - \log_5 3 + \log_5 7$ as a single logarithm.
3. Evaluate $\log_3 14$. Round your answer to three decimal places.
4. Use the properties of logarithms to evaluate $\log_2 \sqrt{32}$.
5. Use the properties of logarithms to evaluate $\log_9 9$
6. State the equation of the asymptote for the function $y = -2\log\left[\frac{1}{3}(x+4)\right] - 1$.
7. Solve $\log_7(x+1) + \log_7(x-5) = 1$.
8. A wound, initially with an area of 40 cm², heals according to the formula $A(t) = 40(10^{-0.023t})$ where $A(t)$ is the area of the wound in square centimeters after t days of healing. In how many days will 60% of the wound be healed? Round your answer to one decimal place.
9. The pH of a substance is given by the equation $pH = -\log[H^+]$ where $[H^+]$ is the hydrogen ion concentration in moles per litre (mol/L). Determine the hydrogen ion concentration in vinegar with a pH of 2.2. Round your answer to four decimal places.

Part D: Characteristics of Functions

1. State whether the function $f(x) = 2x^3 + x$ is even, odd, or neither.
2. State the domain of $y = 2^{x-4} + 5$.
3. State the range of $y = -3\sqrt{x+8} - 4$.
4. State the interval of increase for $y = -2(x+1)^2 - 4$.
5. State the interval of decrease for $y = 4\left|\frac{1}{3}(x-7)\right| + 5$.
6. **True or False:** The average rate of change is represented graphically by the slope of a secant.
7. **True or False:** The minimum point occurs when the slopes of the tangents are positive before the point and the slopes of the tangents are negative after the point.
8. **True or False:** x = 3.99 is an approximate solution of the equation $-3\csc x = x$.
9. State the domain of the function $f(x) = x^2 + \sqrt{x+3}$.
10. State the domain of the function $f(x) = \dfrac{\log x}{|x-2|}$.
11. If $f(x) = \cot x$ and $g(x) = x^2$, determine $f(g(-2))$. Round your answer to three decimal place.
12. Determine the domain and range of the function $f(x) = \sin(2^x)$.

13. A person's blood pressure, P(t), in millimeters of mercury (mm Hg), is modeled by the function $P(t) = 100 - 20\cos\left(\dfrac{8\pi}{3}t\right)$, where t is the time in seconds. Calculate the average rate of change in a person's blood pressure between t = 0.2 seconds and t = 0.3 seconds.

14. Solve $\dfrac{(x^2 - 1)}{\sin x} < 0$ for $0 \le x \le 2\pi$.

ANSWERS

PART A

1. y = kx(x + 2)(x − 3)
2. false
3. true
4. x = -2/3
5. y = 2
6. false
7. (x − 2)(x + 5)(x + 6)
8. $f(x) = -2(x+3)(x+1)(x-5)^2$
9. $0.6 < t < 14.4$

PART B

1. $\dfrac{23\pi}{36}$
2. $\dfrac{1}{\sqrt{3}}$
3. -1.395
4. $\theta = \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$
5. Period = 3π, Amplitude = 2, Phase Shift = $\dfrac{\pi}{6}$, Vertical Displ. = -5
6. $x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \pi$
7. May 12 – August 18
8. $x = \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \dfrac{3\pi}{2}, \dfrac{7\pi}{4}$

PART C

1. 1.362
2. $\log_5 35$
3. 2.402
4. $\dfrac{5}{2}$
5. 1
6. x = -4
7. x = 6
8. 17.3 days
9. 0.0063

PART D

1. odd
2. $\{x \in \mathbb{R}\}$
3. $\{y \in \mathbb{R} \mid y < -4\}$
4. $\{x \in \mathbb{R} \mid x < -1\}$
5. $\{x \in \mathbb{R} \mid x < 7\}$
6. True
7. False
8. True
9. $\{x \in \mathbb{R} \mid x \ge -3\}$
10. $\{x \in \mathbb{R} \mid x > 0, x \ne 2\}$
11. 0.864
12. $D = \{x \in \mathbb{R}\}, R = \{y \in \mathbb{R} \mid -1 \le y < 1\}$
13. 140
14. $x \in (0,1), x \in (\pi, 2\pi)$