Properties of Limits

For any real number a, if **f** and **g** both have limits that exist at $x = a$, the limit may be evaluated using:

1. $\lim_{x \to a} k = k$, for any real constant k
2. $\lim_{x \to a} x = a$
3. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
4. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$
5. $\lim_{x \to a} [f(x)g(x)] = [\lim_{x \to a} f(x)][\lim_{x \to a} g(x)]$
6. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$
7. $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$

Properties of Limits

Feb 7/2018

Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 2} \dfrac{x - 2}{4 - x^2}$

(b) $\lim_{x \to 3} \dfrac{|x - 3|}{x - 3}$

(c) $\lim_{x \to 0} \dfrac{\sqrt{x+3} - \sqrt{3-x}}{x}$

(d) $\lim_{x \to 0} \dfrac{(x+8)^{\frac{1}{3}} - 2}{x}$

Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(a) $$\lim_{x\to2}\frac{x-2}{4-x^2} = \lim_{x\to2}\frac{\cancel{x-2}^{\,1}}{\underset{-1}{\cancel{(2-x)}}(2+x)}$$

$2-x$

$= -x+2$

$= -(x-2)$

$$= \lim_{x\to2}\frac{-1}{2+x}$$

$$= \frac{-1}{4}$$

Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(b) $$\lim_{x\to3}\frac{|x-3|}{x-3}$$

$$f(x) = \frac{|x-3|}{x-3} = \begin{cases}\frac{x-3}{x-3}, & x>3\\ \frac{-(x-3)}{x-3}, & x<3\end{cases}$$

need to break into the one-sided limits

$$\lim_{x\to3^+}\frac{|x-3|}{x-3}$$

$$= \lim_{x\to3^+}\frac{x-3}{x-3}$$

$$= \lim_{x\to3^+}(1)$$

$$= 1$$

$$f(x) = |x| = \begin{cases}x, & x\geq0\\ -x, & x<0\end{cases}$$

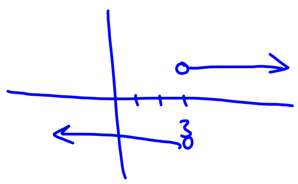
$$\lim_{x\to3^-}\frac{-\cancel{(x-3)}^{\,1}}{\cancel{(x-3)}_{\,1}}$$

$$= \lim_{x\to3^-}(-1)$$

$$= -1$$

∵ one-sided limits are ~~not~~ not equal.

$\therefore \lim_{x\to3}\frac{|x-3|}{x-3}$ DNE



Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(c) $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3-x}}{x} \times \frac{\sqrt{x+3}+\sqrt{3-x}}{\sqrt{x+3}+\sqrt{3-x}}$

$= \lim_{x\to 0} \frac{(x+3)-(3-x)}{x(\sqrt{x+3}+\sqrt{3-x})}$

$= \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(\sqrt{x+3}+\sqrt{3-x})}$

$= \frac{2}{\sqrt{3}+\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

Ex.1 Evaluate each limit, if it exists. If it does not exist, explain why.

(d) $\lim_{x\to 0} \frac{(x+8)^{\frac{1}{3}}-2}{x}$

$= \lim_{u\to 2} \frac{u-2}{u^3-8}$

$= \lim_{u\to 2} \frac{\cancel{(u-2)}}{\cancel{(u-2)}(u^2+2u+4)}$

$= \frac{1}{12}$

* change in variable

let $u=(x+8)^{\frac{1}{3}}$

* must replace all references to x.

$u=(x+8)^{\frac{1}{3}}$

$u^3=x+8$

$x=u^3-8$

as $x\to 0$, $u\to (0+8)^{\frac{1}{3}}$

$=2$

In Summary:

To help evaluate limits we can:

- substitute directly (easy) (not shown)
- factor (as in a)
- rationalize (as in c)
- use one-sided limits (as in b)
- use a change of variable (as in d)

Assigned Work:

p.45 # 4, 7bce, 8cef, 9, 10ad

p.45 8(c)

$$\lim_{x\to 1} \frac{x^{\frac{1}{6}}-1}{x-1}$$

let $u = x^{\frac{1}{6}}$

$u^6 = x$

as $x \to 1$, $u \to 1$

$$= \lim_{u\to 1} \frac{u-1}{u^6-1}$$ simpler

OR

$$= \lim_{u\to 0} \frac{u}{(u+1)^6-1}$$

let $u = x^{\frac{1}{6}} - 1$

$u+1 = x^{\frac{1}{6}}$

$(u+1)^6 = x$

as $x \to 1$, $u \to 0$

① diff. cubes

$(u^2)^3 - 1$

② diff. squares

$(u^3)^2 - 1$

③ long division

$(u^6-1) \div (u-1)$

$u^6 - 1 = (u-1)\,Q$

10 (a) $\lim_{x \to -2} \frac{(x+2)^3}{|x+2|}$

let $f(x) = \frac{(x+2)^3}{|x+2|}$

$$f(x) = \begin{cases} \frac{(x+2)^{\cancel{3}\,2}}{\cancel{-(x+2)}} & x < -2 \\ \frac{(x+2)^{\cancel{3}\,2}}{\cancel{x+2}} & x > -2 \end{cases}$$

$$= \begin{cases} -(x+2)^2 & x < -2 \\ (x+2)^2 & x > -2 \end{cases}$$