Continuity of a Function

The function, $f(x)$, is said to be continuous at $x = a$ if:

1. The limit at $x = a$ exists: $\lim_{x \to a} f(x) = L$

(note: both left and right side limits must exist and be equal)

2. The function value at $x = a$ is defined: $f(a) = \text{L}$

As a single statement:

$$\lim_{x \to a} f(x) = f(a)$$

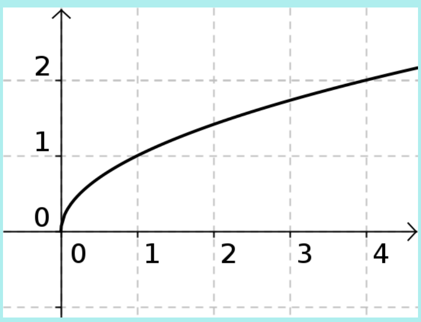
Note: All polynomial functions are continuous for all real numbers.

What about limited domains?

(1) Some functions have restrictions on their domain.
(2) In some cases, we are only interested in a limited part of a domain.

For example, the radical function, $f(x) = \sqrt{x}$

$D_f = x \in [0, \infty)$



Is it continuous at x = 0?

$\lim_{x \to 0^-} \sqrt{x}$ DNE

$\lim_{x \to 0^+} \sqrt{x} = 0 = f(0)$

Yes. Since x = 0 is the left-most value in the domain, we only consider the right-hand limit.

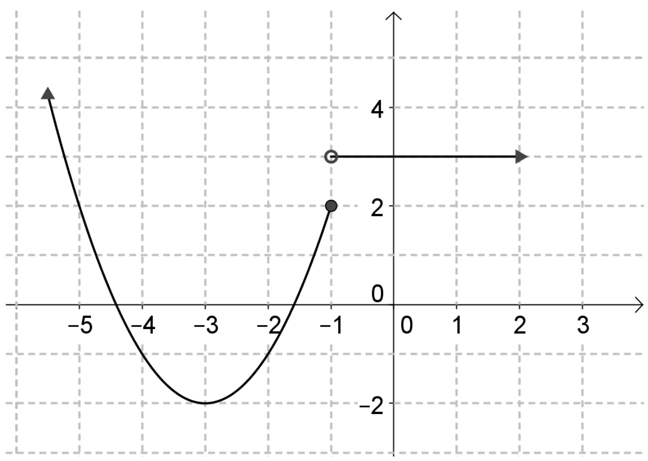
A function which is not continuous has a discontinuity.

There are three types of discontinuity:

(a) a jump discontinuity (piecewise function)
(b) a removable discontinuity (hole)
(c) an infinite discontinuity (vertical asymptote)

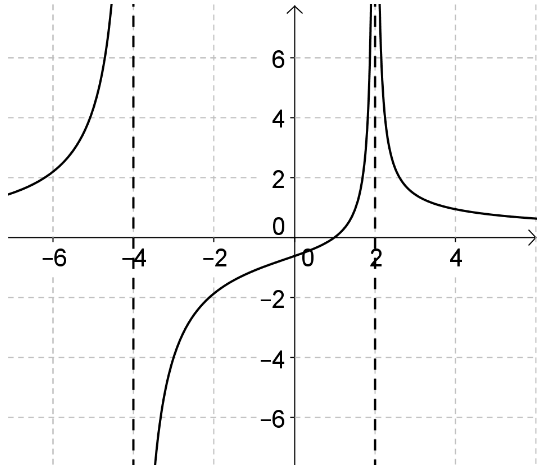
Ex.1 Identify where each of the following functions is discontinuous, and state the type of discontinuity.

(a)



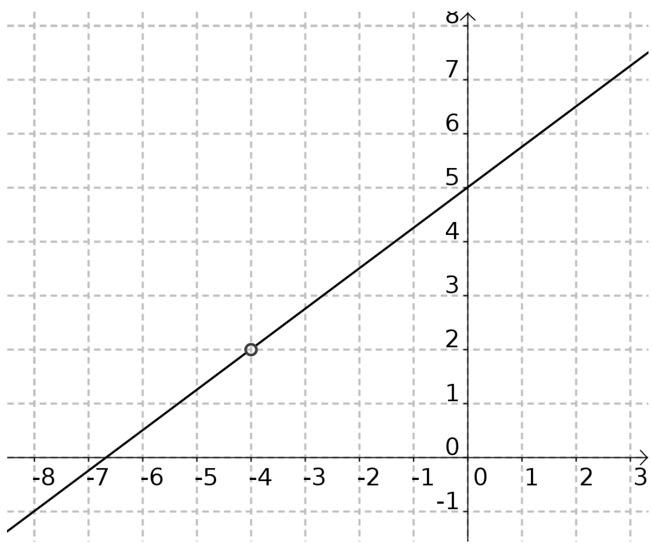
Ex.1 Identify where each of the following functions is discontinuous, and state the type of discontinuity.

(b)



Ex.1 Identify where each of the following functions is discontinuous, and state the type of discontinuity.

(c)



Assigned work:

Read summary on p.51

p.52 # 4ace, 5bcdf, 7, 8, 12, 14, 15

15. $f(x) = \begin{cases} \frac{Ax-B}{x-2} & x \leq 1 \\ 3x & 1 < x < 2 \\ Bx^2 - A & x \geq 2 \end{cases}$

cont @ $x = 1$

$$\lim_{x \to 1} f(x) = f(1)$$

$\because$ no holes or VA @ $x = 1$

$$\frac{A(1)-B}{1-2} = 3(1)$$

$$B - A = 3$$

$$B - 3 = A$$

discontinuous @ $x = 2$

$$3x \neq Bx^2 - A$$

$$3(2) \neq B(2)^2 - A$$

$$6 \neq 4B - A$$

$$6 \neq 4B - (B-3)$$

$$6 \neq 3B + 3$$

$$3 \neq 3B$$

$$B \neq 1$$

$$\Rightarrow A \neq -2$$