Derivatives - The Power Rule

Feb 9/2018

Recall, at $x = a$:

$$m_{\text{tangent}} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Now, the derivative of $f(x)$ at $x = a$:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

or

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

The Power Rule:

Given $f(x) = x^n$, where $n \in \mathbb{Z}$

then $f'(x) = nx^{n-1}$

where $f'(x)$ is (a) the slope of the tangent at x
or (b) the instantaneous RoC at x

Ex.1 Find the derivative of each function.

Hint: Express all terms as powers of x

(a) $y = 3x^2 - 5x + 7$

(b) $y = x(x-4)(x+4)$

(c) $y = \dfrac{5}{x^2}$

(d) $y = \sqrt{2x^3}$

Notation used for the derivative of a function:

y' " y prime "

$\frac{dy}{dx}$ the differential of y with respect to the differential of x

$f'(x)$ " f prime of x "

$\frac{d}{dx}f(x)$ Leibniz notation: the derivative of $f(x)$ with respect to x

$\frac{d}{dx}(5x^3 - 4x + 3) =$ ______

Constant Rule: If $f(x) = k$ then $f'(x) = 0$

$\boxed{\text{Power Rule: If } f(x) = x^n \text{ then } f'(x) = nx^{n-1}}$

Constant Multiple Rule: If $f(x) = kg(x)$ then $f'(x) = kg'(x)$

Sum Rule: If $f(x) = g(x) + h(x)$, then $f'(x) = g'(x) + h'(x)$

Difference Rule: If $f(x) = g(x) - h(x)$, then $f'(x) = g'(x) - h'(x)$

Constant Rule: $\frac{d}{dx}[k]=0$

Power Rule: $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$

Constant Multiple Rule: $\frac{d}{dx}[kf(x)]=k\frac{d}{dx}[f(x)]$

Sum Rule: $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$

Difference Rule: $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$

Ex.1 Find the derivative of each function.

Hint: Express all terms as powers of x

(a) $y=3x^2-5x+7$ (b) $y=x(x-4)(x+4)$

$y'=2(3x^{2-1})-1(5x^{1-1})+0$

$y'=6x-5$

(b) $y=x(x^2-16)$

$=x^3-16x$

$y'=3x^2-16$

Ex.1 Find the derivative of each function.

Hint: Express all terms as powers of x

(c) $y = \dfrac{5}{x^2}$

$y = 5x^{-2}$

$y' = -10x^{-3}$

or

$y' = \dfrac{-10}{x^3}$

(d) $y = \sqrt{2x^3}$

$y = \sqrt{2}\, x^{\frac{3}{2}}$

$y' = \frac{3}{2}\sqrt{2}\, x^{\frac{1}{2}}$

$= \dfrac{3\sqrt{2}}{2} x^{\frac{1}{2}}$

or

$= \dfrac{3\sqrt{2x}}{2}$

Ex.2 Find the point on the curve $y = x^2 + 2x - 3$ where the slope of the tangent is equal to 2. Sketch this situation.

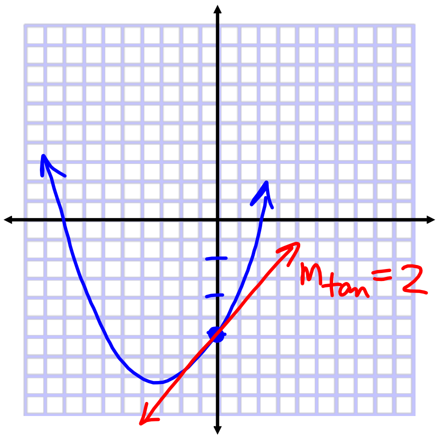
want $y' = 2$

$2x + 2 = 2$

$2x = 0$

$x = 0$

slope of tangent is 2 when $x = 0$



when $x = 0$, $y = (0)^2 + 2(0) - 3$
$= -3$

$\therefore m_{tan} = 2$ at $P(0, -3)$

Assigned Work:

summarize table on p.81 into your notes
read proofs on p.76, p.77
p.82 # 2, 3def, 4cef, 8ab, 9bf, 14, 15, 16

3(f) $s(t) = \frac{t^5 - 3t^2}{2t}$ $\qquad x^n$

$= \frac{1}{2}t^{-1}(t^5 - 3t^2)$

$s(t) = \frac{1}{2}t^4 - \frac{3}{2}t^1$

$s'(t) = 2t^3 - \frac{3}{2}$

9(b) $y = \frac{3}{x^2} - \frac{4}{x^3}$

$= 3x^{-2} - 4x^{-3}$

$\frac{dy}{dx} = -6x^{-3} + 12x^{-4}$ @ $P(-1,7)$

slope of tangent

$m = \frac{-6}{(-1)^3} + \frac{12}{(-1)^4}$

$= 6 + 12$

$= 18$

$y = 18x + b$, sub $P(-1,7)$

$7 = 18(-1) + b$

$b = 25$

$\boxed{y = 18x + 25}$

9(f) $y = \frac{\sqrt{x} - 2}{\sqrt[3]{x}}$ $P(1,-1)$

$= x^{-\frac{1}{3}}(x^{\frac{1}{2}} - 2)$

$= x^{\frac{1}{6}} - 2x^{-\frac{1}{3}}$

$y' = \frac{1}{6}x^{-\frac{5}{6}} + \frac{2}{3}x^{-\frac{4}{3}}$

m, sub $x = 1$

$\vdots$

15. $y = x^3 + 2$

where is $m_{tan} = 12$?

$y' = 12$?

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

Sub to find y-coord

16. $y = \frac{1}{5}x^5 - 10x$

$y' = x^4 - 10$

want

$x^4 - 10 = 6$

$x^4 = 16$

$x = \pm 2 \leftarrow$ two points where $m = 6$

$\therefore$ two tangents