Oblique Asymptotes

Oblique asymptotes occur in rational functions when the degree of the numerator is greater than the degree of the denominator. For a difference of one, the OA is linear.

To determine an equation for the asymptote, divide (long or synthetic) the denominator into the numerator.

Consider the limiting behaviour, removing terms where their contribution to the limit becomes zero.

Ex.1 Determine equation(s) and end behaviour(s) for oblique asymptotes of

$$f(x) = \frac{2x^2+3x-1}{x+1}$$

$O_N = 2$

$O_D = 1$

Ex.1 Determine ① equation(s) and ② end behaviour(s) for oblique asymptotes of

$$f(x) = \frac{2x^2+3x-1}{x+1}$$

①

$$\begin{array}{r|l} & \ \ 2x + 1 \\ x+1 & 2x^2 + 3x - 1 \\ & \underline{2x^2 + 2x} \ \downarrow \\ & \qquad x - 1 \\ & \qquad \underline{x + 1} \\ & \qquad\ -2 \ R \end{array}$$

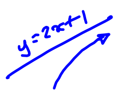
$$= \underbrace{2x+1}_{\text{OA: } y = 2x+1} - \underbrace{\frac{2}{x+1}}_{\text{end behaviour}}$$

② $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left[\underbrace{2x+1}_{\text{OA (ignore)}} - \frac{2}{x+1}\right]$

$\frac{2}{x+1} \to$ sm. num ($x+1 \to \infty$)

$= \underbrace{2x+1} -$ sm. num.

OA: $y = 2x+1 \to$ y-coordinate

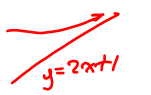


as $x \to \infty$, $f(x) \to 2x+1$ from below.

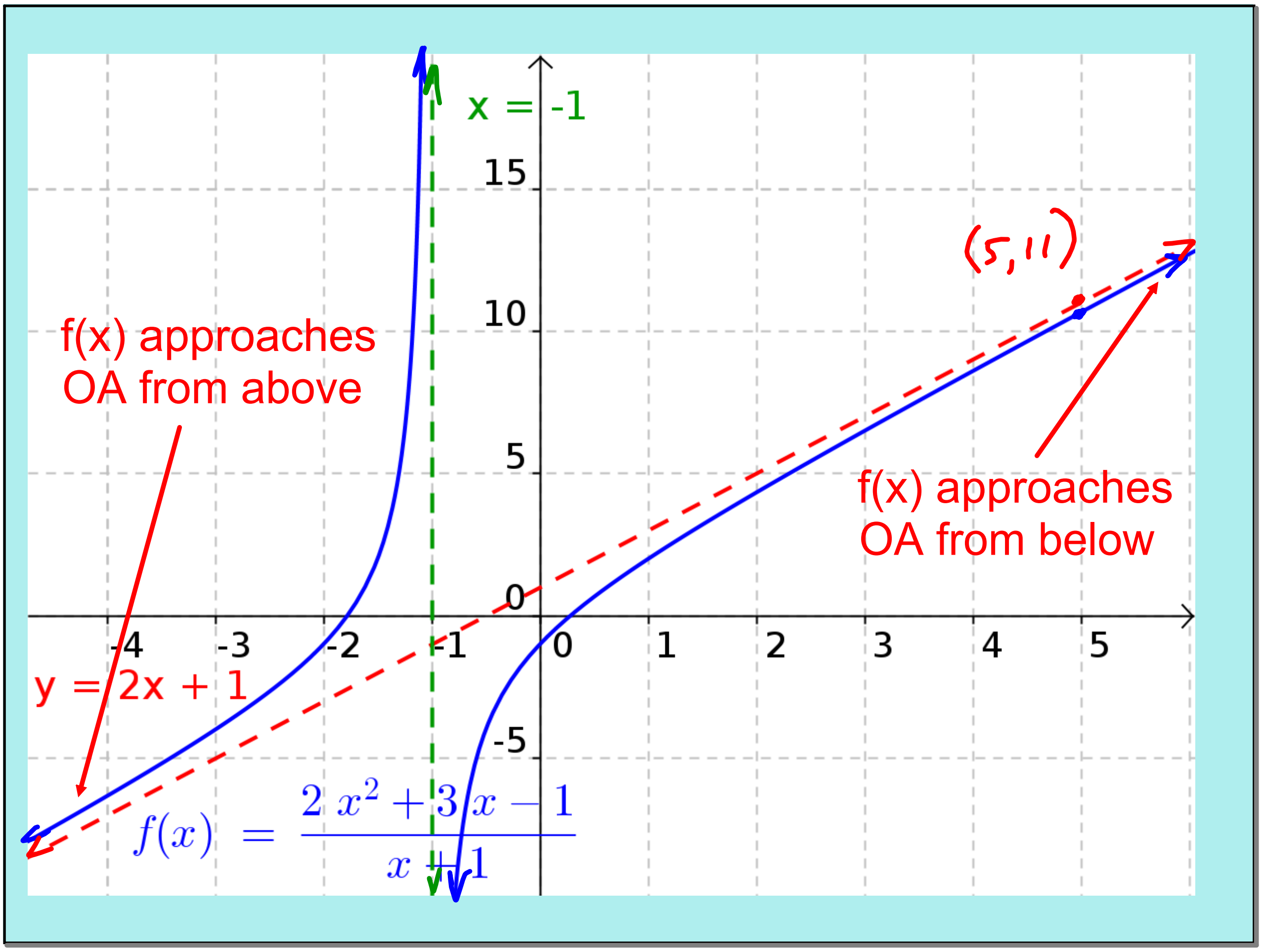
$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left[\underbrace{2x+1}_{\text{OA}} - \frac{2}{x+1}\right]$

$\frac{2}{x+1} \to$ neg. sm. number ($x+1 \to -\infty$)

$= 2x+1 +$ sm. num.



as $x \to -\infty$, $f(x) \to 2x+1$ from above.



Assigned Work:

p.193 # 3d, 4d, 5d, 6c, 7, 9d, 10, 14

$$\lim_{x \to \pm\infty} f(x)$$