Critical Points & 1st Derivative Test

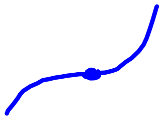
If f '(a) = 0, then x = a is a $\underbrace{\text{critical value}}_{x\text{-only}}$ and (a, f(a)) is a critical point on f(x).

x y

A critical point can be

f goes from INC to DEC

- a maximum (slope changes from + to -)
- a minimum (slope changes from - to +)
- a point of inflection (trend in slope continues)



PoI

If f '(a) is undefined, then x = a is a critical value .

On f(x), this critical value can occur at

- a vertical asymptote (no critical point, a VA for f(x))
- a max or a min (the critical point is a cusp)

Ex.1 Consider the function defined by $f(x) = x^4 - 3x^3$.

a) Find all intercepts.

b) Find all critical points.

(a) y-int : $f(0) = 0^4 - 3(0)^3$

$= 0$

$(0,0)$

x-int : $f(x) = 0$

$x^4 - 3x^3 = 0$

$x^3(x-3) = 0$

$x = 0$ or $x = 3$

triple root   linear root

(b) $f'(x) = 4x^3 - 9x^2$

set $f'(x) = 0$

$0 = x^2(4x-9)$

$x = 0$ or $4x - 9 = 0$

$x = \frac{9}{4}$

critical values

$f(0) = 0$

$\downarrow$

$CP_1\ (0,0)$

$f\left(\frac{9}{4}\right) = \left(\frac{9}{4}\right)^4 - 3\left(\frac{9}{4}\right)^3$

$= \frac{6561}{256} - 3\left(\frac{729}{64}\right)$

$= \frac{-2187}{256}$

$CP_2\ \left(\frac{9}{4}, \frac{-2187}{256}\right)$

Ex.1 Consider the function defined by $f(x) = x^4 - 3x^3$.
c) Classify the critical points using an INC/DEC table.

$$\begin{aligned} f'(x) &= 4x^3 - 9x^2 \\ 0 &= x^2(4x-9) \end{aligned}$$

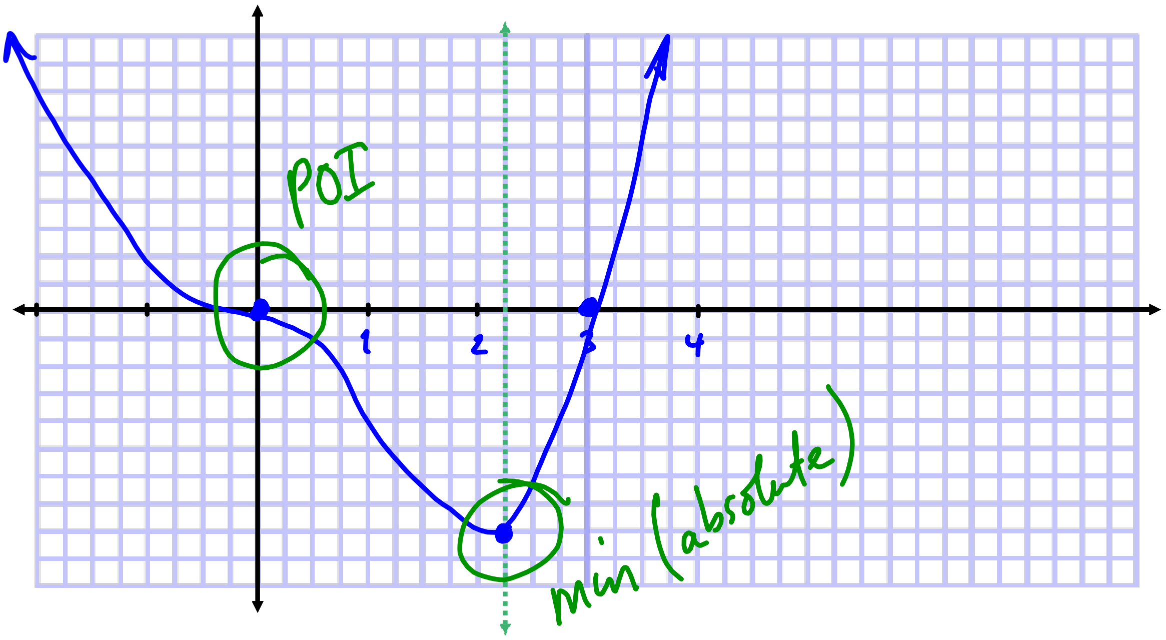
-1 0 1 $\frac{9}{4}$ 5

interval	$(-\infty, 0)$	$(0, \frac{9}{4})$	$(\frac{9}{4}, \infty)$
x^2	+	+	+
$4x-9$	−	−	+
sign of $f'(x)$	−	−	+
INC/DEC?	DEC ↘	DEC ↘	INC ↗

trend continues
PoI at $x=0$
$P(0,0)$

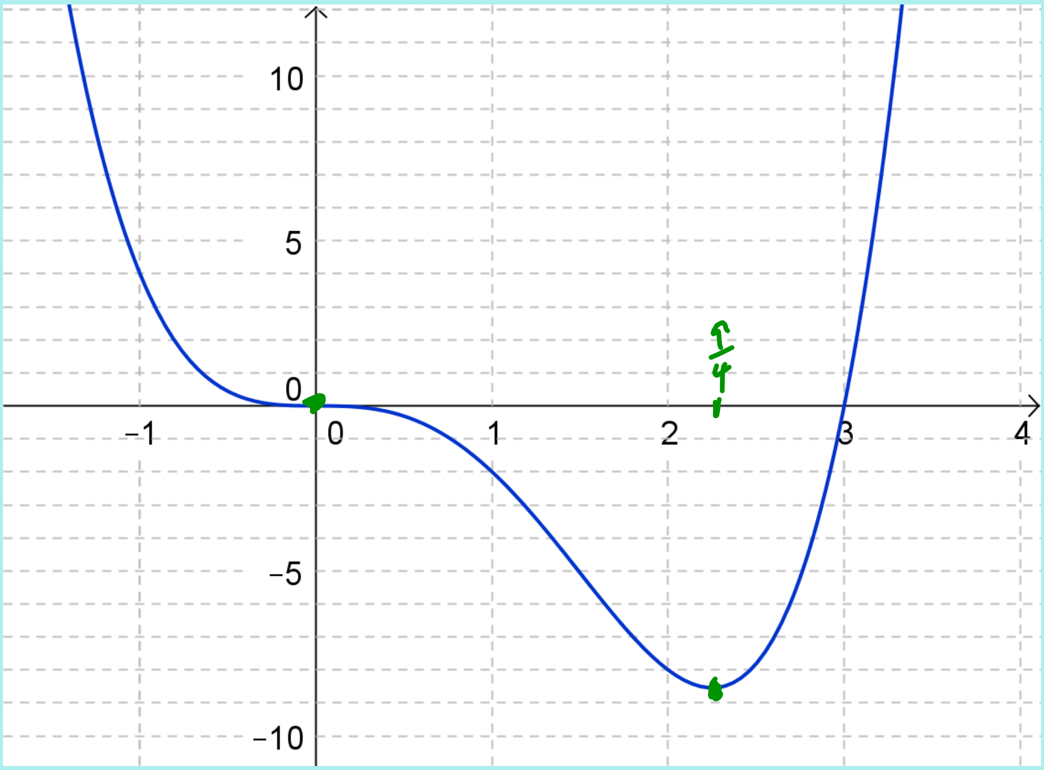
local min
at
$\left(\frac{9}{4}, -\frac{2187}{256}\right)$

Ex.1 Consider the function defined by $f(x) = x^4 - 3x^3$.
d) Sketch $f(x)$.



Ex.1 Consider the function defined by $f(x) = x^4 - 3x^3$.

d) Graph $f(x)$.



Ex.2 Find the critical point(s) of the function

$$f(x) = \sqrt[5]{(x+3)^2}$$

$$= \left[(x+3)^2\right]^{\frac{1}{5}}$$

$$= (x+3)^{\frac{2}{5}}$$

$$f'(x) = \frac{2}{5}(x+3)^{-\frac{3}{5}}(1)$$

for critical values, set $f'(x)=0$

$$0 = \frac{2}{5\sqrt[5]{(x+3)^3}}$$

cross multiply

$0 = 2$???

$\therefore$ no solution

also : $5\sqrt[5]{(x+3)^3} \neq 0$

$x+3 \neq 0$

$x \neq -3$

in $f'(x)$

but $f(-3) = \sqrt[5]{(-3+3)^2}$

$= 0$

$f(-3)$ exists

$f'(-3)$ ~~DNE~~ undefined

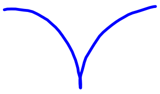
↓

$x = -3$ is a critical value

test $x \to -3^-$, $x \to -3^+$

$f(-2) = 1$

$f(-4) = 1$



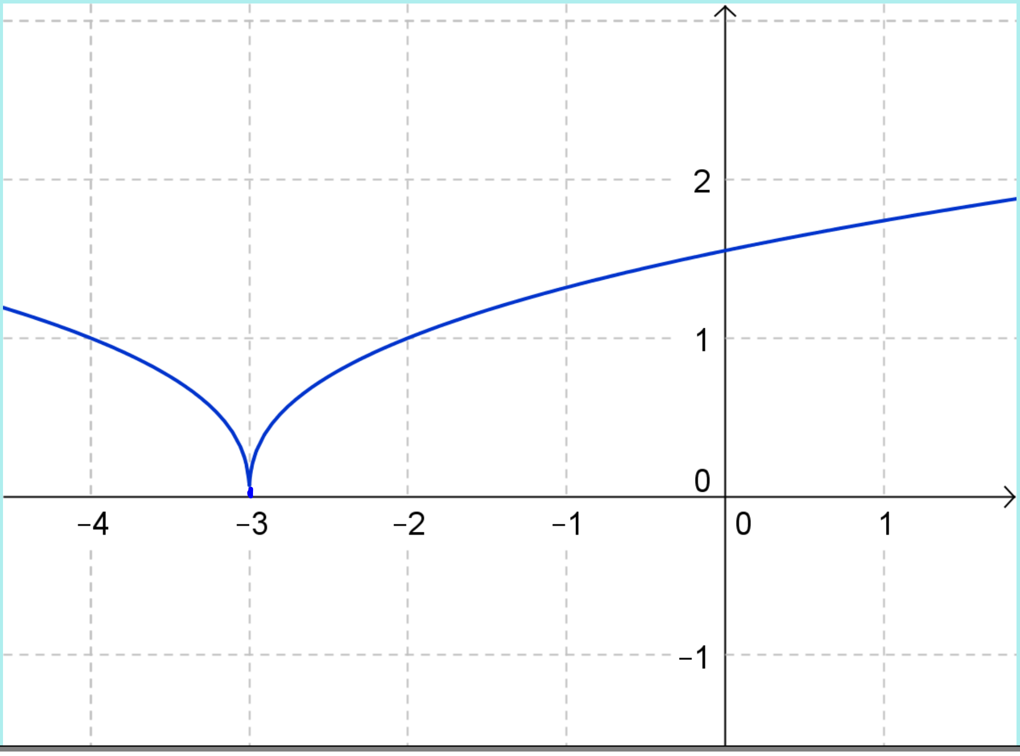
$(-3,0)$ is a critical point

(a cusp)

$\therefore (-3,0)$ is a minimum.

Ex.2 Find the critical point(s) of the function

$$f(x)=\sqrt[5]{(x+3)^2}$$



Assigned Work:

p.178 # 3, 4, 5bcd, 7cdef