

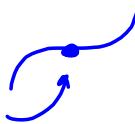
Critical Points & 1st Derivative Test

Feb 26/2018

If  $f'(a) = 0$ , then  $x = a$  is a **critical value** and  $(a, f(a))$  is a **critical point** on  $f(x)$ .

A critical point can be

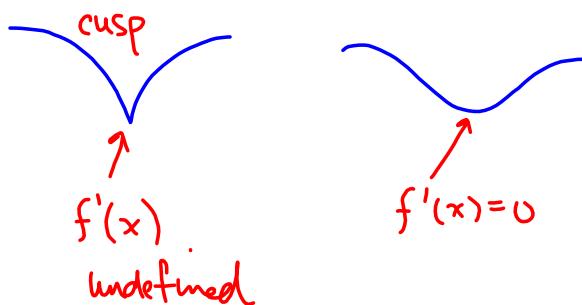
- a maximum (slope changes from + to -)
- a minimum (slope changes from - to +)
- a point of inflection (trend in slope continues)



If  $f'(a)$  is undefined, but  $f(a)$  is defined, then  $x = a$  is a **critical value**.

On  $f(x)$ , this critical value can occur at

- a vertical asymptote (no critical point, a VA for  $f(x)$ )
- a max or a min (the **critical point** is a cusp)



Ex.1 Consider the function defined by  $f(x) = x^4 - 3x^3$ .

- Find all intercepts.
- Find all critical points.

(a)  $y\text{-int} : f(0) = 0$

$x\text{-int} : \text{set } f(x) = 0$   
 $0 = x^3(x-3)$

$x=0, x=3$

(b)  $f'(x) = 4x^3 - 9x^2$

Set  $f'(x) = 0$   
 $0 = x^2(4x-9)$

$x=0, x=\frac{9}{4}$  → these are critical  
values

$$\begin{aligned} f(0) &= 0 & f\left(\frac{9}{4}\right) &= \left(\frac{9}{4}\right)^4 - 3\left(\frac{9}{4}\right)^3 \\ &= 0 & &= \frac{6561}{256} - \frac{2187}{64} \end{aligned}$$

∴ critical  
 points are

$$(0,0) \text{ and } \left(\frac{9}{4}, \frac{2187}{256}\right)$$

Ex.1 Consider the function defined by  $f(x) = x^4 - 3x^3$ .

c) Classify the critical points using an INC/DEC table.

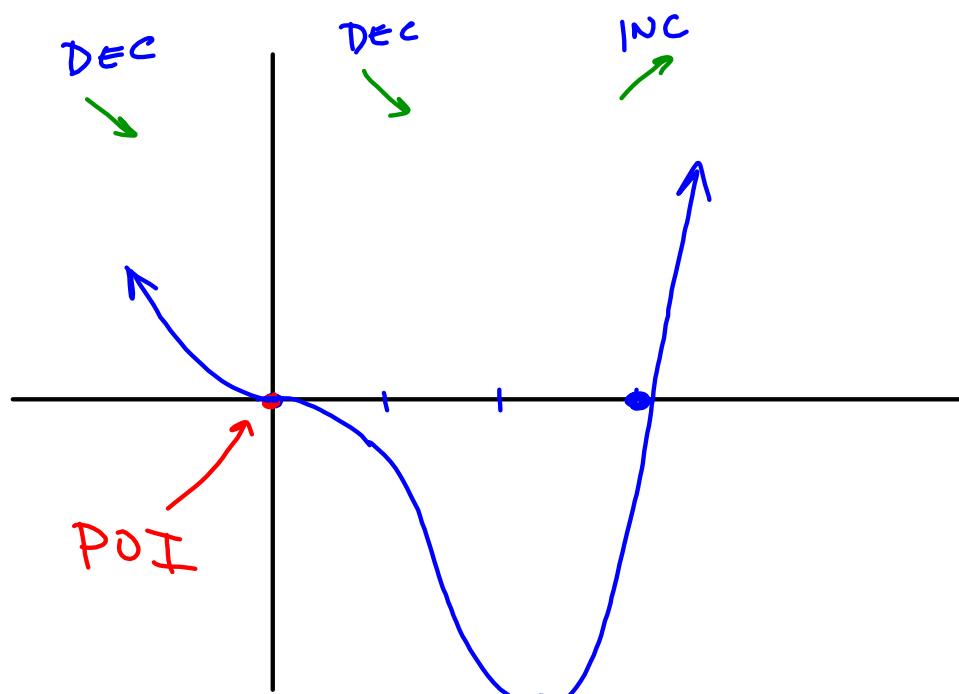
$$\begin{aligned} f'(x) &= 4x^3 - 9x^2 \\ &= x^2(4x - 9) \end{aligned}$$

i.e., an interval table using  $f'(x)$

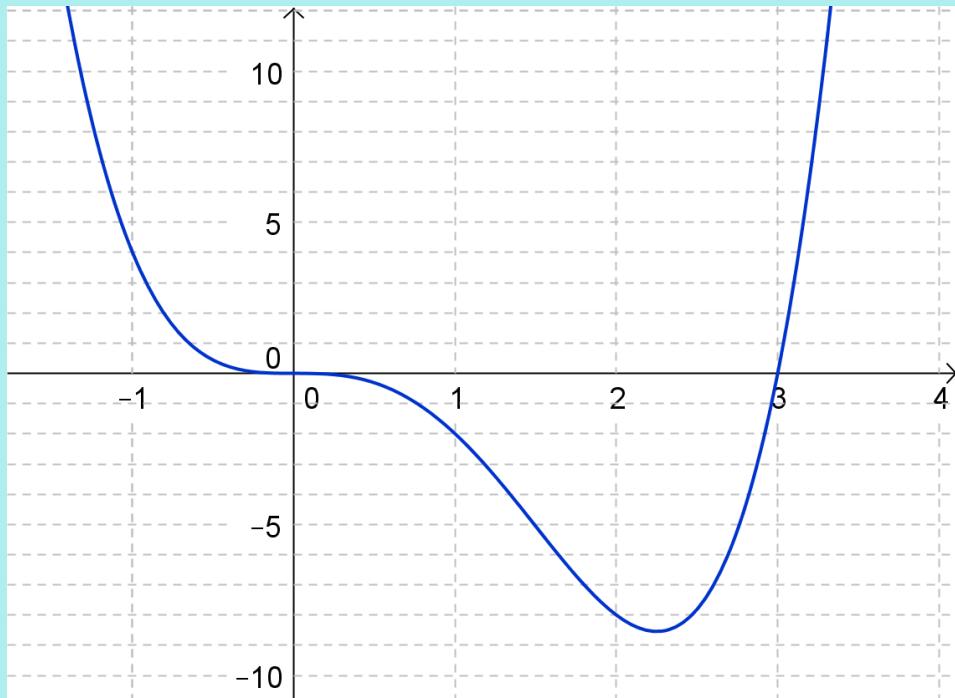
interval	$(-\infty, 0)$	$(0, \frac{9}{4})$	$(\frac{9}{4}, \infty)$
$x^2$	+	+	+
$4x - 9$	-	-	+
sign of $f'(x)$	-	-	+
INC/DEC?	$f$ is DEC	$f$ is DEC	$f$ is INC

Ex.1 Consider the function defined by  $f(x) = x^4 - 3x^3$ .

d) Sketch  $f(x)$ .



Ex.1 Consider the function defined by  $f(x) = x^4 - 3x^3$ .  
d) Graph  $f(x)$ .

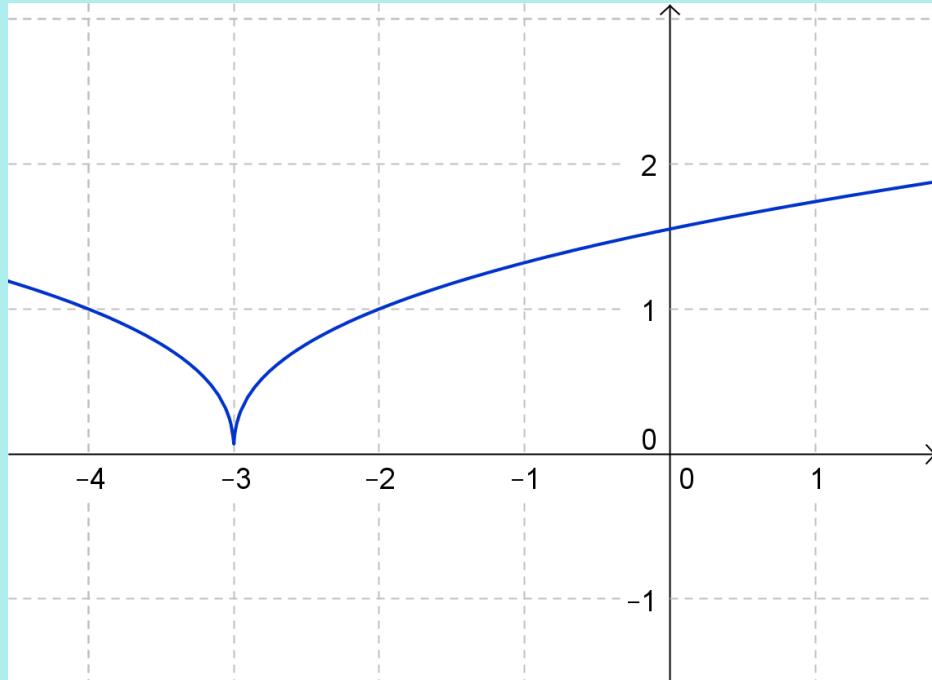


Ex.2 Find the critical point(s) of the function

$$f(x) = \sqrt[5]{(x+3)^2}$$

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Assigned Work:

p.178 # 3, 4a, 5b, 7c, d, f

4(b)  $f(x) = \frac{2x}{x^2+9}$

$x\text{-int: } x=0$   
 $y\text{-int: } y=0$

$$f'(x) = \frac{2(x^2+9) - 2x(2x)}{(x^2+9)^2}$$

$$= \frac{-2x^2+18}{(x^2+9)^2}$$

Set  $f'(x)=0$

$$0 = \frac{-2x^2+18}{(x^2+9)^2} = \frac{-2(x-3)(x+3)}{(x^2+9)^2}$$

$$0 = -2(x^2-9)$$

$$x^2 = 9$$

$$x = \pm 3 \rightarrow \text{critical values}$$

$f'(x)$

-3	+3
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DEc min InE max DEc

Graph showing the function  $f(x) = \frac{2x}{x^2+9}$  with a local minimum at  $(-3, -\frac{2}{9})$  and a local maximum at  $(3, \frac{2}{9})$ .

$$5(a) \quad y = (x-5)^{\frac{1}{3}}$$

$$y' = \frac{1}{3}(x-5)^{-\frac{2}{3}}$$

C.V. set  $y' = 0$

$$0 = \frac{1}{3(x-5)^{\frac{2}{3}}} \quad \text{no solution}$$

$y'$  undefined at  $x=5$   
 at  $x=5, y = (5-5)^{\frac{1}{3}} = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} x=5 \text{ is a C.V.}$

$\therefore (5, 0)$  is a C.P.

for  $M_{tan}$  at  $x=5$ ,

Sub  $x=5$  into  $y' \rightarrow$  undefined  
 at  $x=5$

$\therefore M_{tan} \neq 0$

$\therefore$  the tangent is not parallel  
 to the  $x$ -axis.

$$7(d) \quad f(x) = -3x^3 - 5x = x(-3x^2 - 5)$$

$$f'(x) = -9x^2 - 5 \quad \text{set } f'(x) = 0$$

$$0 = x(-3x^2 - 5)$$

for C.V., set  $f'(x) = 0 \quad x = 0$

$$0 = -9x^2 - 5$$

$$-\frac{5}{9} = x^2$$

$$x = \pm \sqrt{-\frac{5}{9}} \quad \text{no solution}$$

always  $< 0$

$f(x)$  is always decreasing

