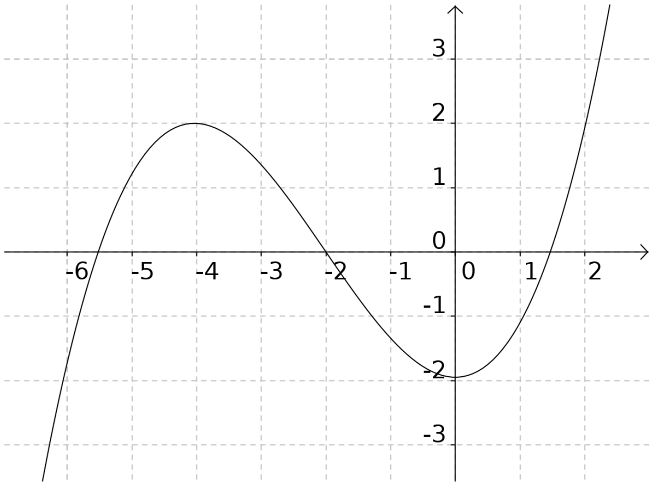
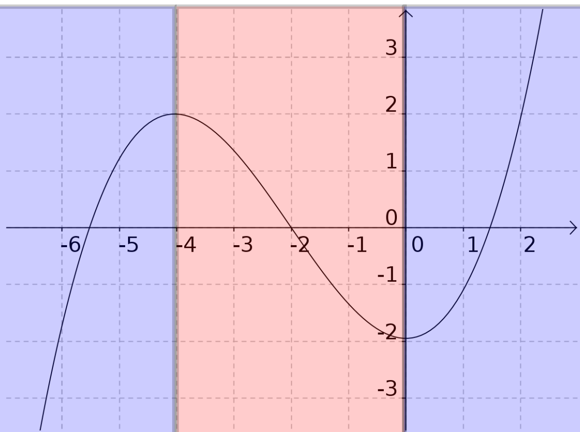
Unit 3 - Curve Sketching

Increasing & Decreasing Functions

On which interval(s) is the function increasing?

On which interval(s) is the function decreasing?





All of the tangent lines in this interval have ___________ slopes.

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All of the tangent lines in this interval have ___________ slopes.

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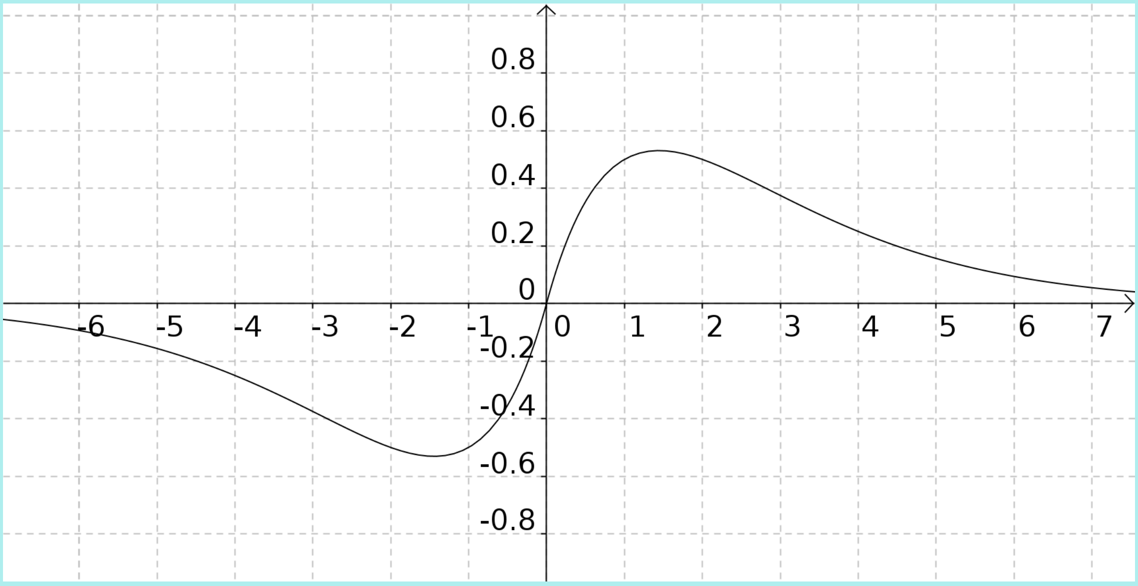
On which intervals is the function decreasing?



Using the derivative to reason about intervals of increase & decrease of a function, we can conclude:

- A function, $f(x)$, is increasing on the interval (a, b) if $f'(x) > 0$.
- A function, $f(x)$, is decreasing on the interval (a, b) if $f'(x) < 0$.

Note: The function <u>must be</u> continuous and differentiable on the interval.

A function is <u>differentiable</u> at any x = a if:

$$f'(a) \text{ exists}$$

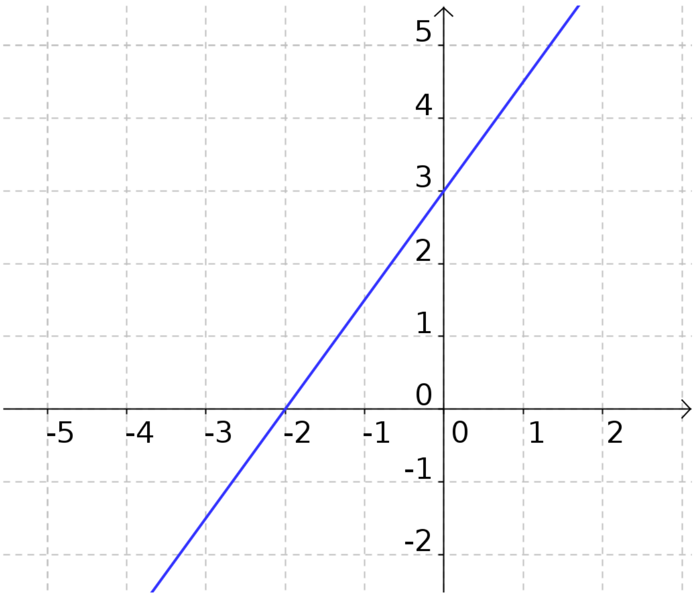
Ex.1 Find the intervals of increase and decrease of the function: $f(x) = 3x^4 + 4x^3 - 12x^2$

Ex.2 Given the graph of $f'(x)$

a) find the intervals of increase and decrease of $f(x)$

b) find the x-value(s) of the local extrema.

c) sketch $f(x)$ assuming $f(0) = 1$



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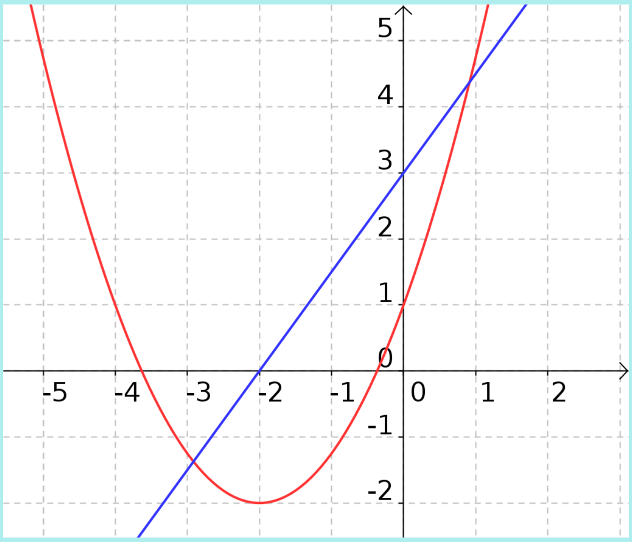
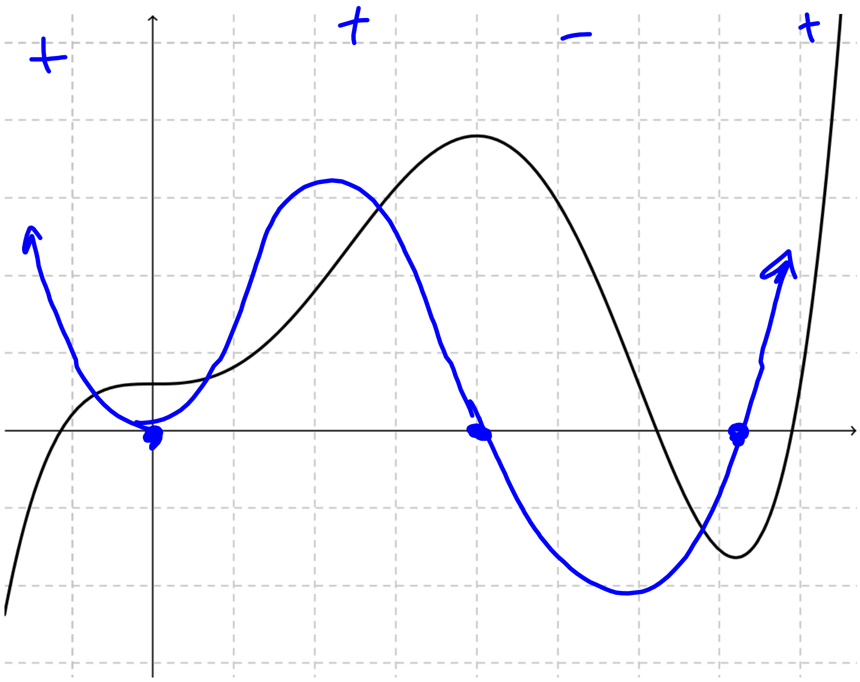


Ex.3 Given the graph of $y = f(x)$ sketch $f'(x)$



Assigned Work:

p.169 # 1ab, 3, 4abcd (algebraically), 8, 9, 10, 11

4. (d) $f(x) = \frac{x-1}{x^2+3}$

$f(x)$ INC/DEC ?

$f'(x)$ +/- ?

$$f'(x) = \frac{(1)(x^2+3)-(x-1)(2x)}{(x^2+3)^3}$$

$$= \frac{x^2+3-2x^2+2x}{(x^2+3)^2}$$

$$= \frac{-x^2+2x+3}{(x^2+3)^2}$$

$$= \frac{-[x^2-2x-3]}{(x^2+3)^2}$$

$$= \frac{-(x-3)(x+1)}{(x^2+3)^2}$$

Set $f'(x)=0$

$$0 = \frac{-(x-3)(x+1)}{(x^2+3)^2}$$

$$x = 3, -1$$

$f'(x)$		-1		3	
-1	$-$		$-$		$-$
$x+1$	$-$		$+$		$+$
$x-3$	$-$		$-$		$+$
$(x^2+3)^2$	$+$		$+$		$+$
	$-$		$+$		$-$
	$f(x)$ DEC		$f(x)$ INC		$f(x)$ DEC

10. $f(x) = ax^2+bx+c$

$$f'(x) = 2ax+b$$

Set $f'(x)=0$

$$0 = 2ax+b$$

$$x = \frac{-b}{2a}$$

$\frac{-b}{2a}+1$ is to the right

$$f'\left(\frac{-b}{2a}+1\right) = 2a\left(\frac{-b}{2a}+1\right)+b$$

$$= -b+2a+b$$

$$= 2a$$

$$> 0$$

11. $f(x) = x^4 - 32x + 4$

$f'(x) = 4x^3 - 32$

$0 = 4x^3 - 32$

$0 = 4(x^3 - 8)$

$0 = 4(x-2)(x^2 + 2x + 4)$

$x = 2$