Oblique Asymptotes

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Oblique asymptotes occur in rational functions when the degree of the numerator is greater than the degree of the denominator. For a difference of one, the OA is linear.

To determine an equation for the asymptote, divide (long or synthetic) the denominator into the numerator.

Consider the limiting behaviour, removing terms where their contribution to the limit becomes zero.

Ex.1 Determine equation(s) and end behaviour(s) for oblique asymptotes of

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$$\begin{array}{r} 2x + 1 \\ x+1 \overline{) 2x^2 + 3x - 1} \\ \underline{2x^2 + 2x} \quad \downarrow \\ x - 1 \\ \underline{x + 1} \\ -2 \end{array} \rightarrow R = -2$$

$$f(x) = \underbrace{2x + 1}_{\text{OA}} - \frac{2}{x+1}$$

OA
$y = 2x + 1$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty}\left(2x + 1 - \frac{2}{x+1}\right)$$

factor highest power of x

$$= \lim_{x \to \infty}\left(2x + 1 - \frac{x\left(\frac{2}{x}\right)}{x\left(1 + \frac{1}{x}\right)}\right)$$

$$= \underbrace{\lim_{x \to \infty}(2x+1)}_{\text{approaching the OA}} - \underbrace{\lim_{x \to \infty}\left(\frac{\cancel{x}\left(\frac{2}{x}\right)}{\cancel{x}\left(1 + \frac{1}{x}\right)}\right)}_{\text{how we approach OA}}$$

$= \text{OA} - \text{sm.pos}$

$= \text{OA}^-$

$= (2x+1)^-$

$$\lim_{x \to -\infty} f(x) = \text{OA} - \lim_{x \to -\infty}\left(\frac{\frac{2}{x}}{1 + \frac{1}{x}}\right)$$

sm.neg

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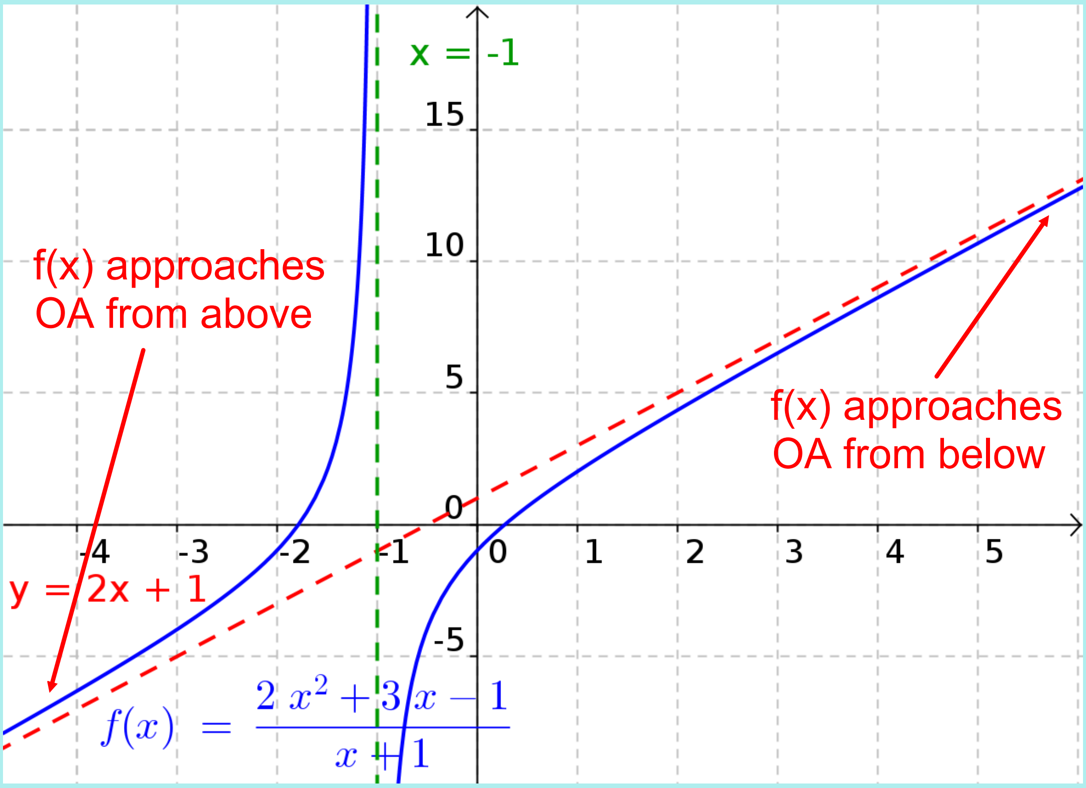
$= \text{OA} - \text{sm.neg}$

$= \text{OA}^+$



Assigned Work:

p.193 # 3d, 4d, 5d, 6c, 7, 9d, 10, 14
c
d
c

10 (c) $y = \frac{20}{x^2+4}$

① intercepts
② asymptotes/holes
③ INC/DEC
④ end behaviour
critical values

① no x-ints
P(0,5)

② no VA
HA: $y=0$

$\lim_{x\to\infty} \frac{20}{x^2+4} = 0^+$ $\lim_{x\to-\infty} \frac{20}{x^2+4} = 0^+$

③ $y = 20(x^2+4)^{-1}$

$y' = 20\left[-(x^2+4)^{-2}(2x)\right]$

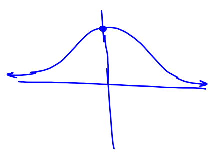
$= \frac{-40x}{(x^2+4)^2}$

CV: $y'=0$ $0=-40x$
$x=0$

CP: $y\Big|_{x=0} = 5$ P(0,5)

y evaluated at $x=0$

y'		$x=0$
-40	$-$	$-$
x	$-$	$+$
$(x^2+4)^2$	$+$	$+$
	$+$	$-$
	INC	DEC



10(d) $s(t) = t + \frac{1}{t}$ ← already in form after long division

set $s=0$, $0 = t + \frac{1}{t}$

$-t = \frac{1}{t}$

$-t^2 = 1$

$0 = 1 + t^2$, no sol.

OA : $s(t) = t$

~~y-int~~ s-int : set $t=0$, but $t \neq 0$

$\therefore$ no s-int

VA : $t = 0$

$\lim_{t\to0^+}\left(\overset{\text{sm}+}{\cancel{t}} + \underbrace{\frac{1}{t}}_{+\infty}\right) = +\infty$ $\lim_{t\to0^-}\left(\overset{\text{sm.neg}}{\cancel{t}} + \underbrace{\frac{1}{t}}_{-\infty}\right) = -\infty$

$\underbrace{\lim_{t\to\infty}}_{\text{OA}}\left(t + \frac{1}{t}\right)$ $\lim_{t\to-\infty}\left(t + \frac{1}{t}\right)$

$=$ OA + sm.pos $= \text{OA} + \text{sm.neg}$

$= \text{OA}^+$ $= \text{OA}^-$

critical values

$s'(t) = 1 - \frac{1}{t^2} = \frac{t^2-1}{t^2} = \frac{(t-1)(t+1)}{t^2}$

$s'(t) = 0$

$0 = 1 - \frac{1}{t^2}$

$\frac{1}{t^2} = 1$

$1 = t^2$

$t = \pm 1$

also, $s'(t)$ is undefined at $t=0$

$\because s(0)$ is also undefined

$\therefore t=0$ is <u>not</u> a CV.

$s(1) = 1 + \frac{1}{1}$
$= 2$

$s(-1) = -1 + \frac{1}{-1}$
$= -2$

CP : $(1,2)$ and $(-1,-2)$

$s'(t)$		-1		0		1	
$t-1$	$-$		$-$		$-$		$+$
$t+1$	$-$		$+$		$+$		$+$
t^2	$+$		$+$		$+$		$+$
	$+$ INC		$-$ DEC		$-$ DEC		$+$ INC