Optimization Problems

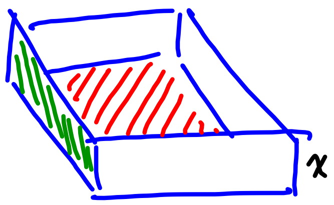
Optimization:

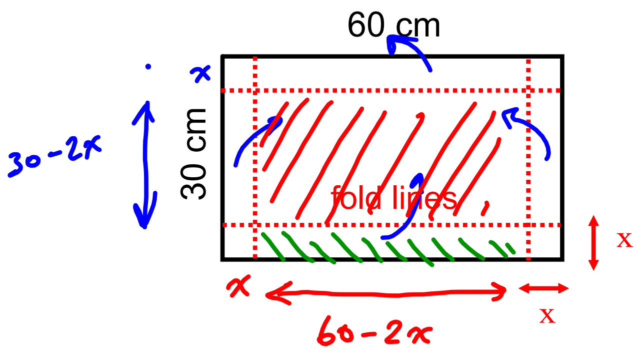
To realize the best possible outcome, subject to a set of restrictions.

$\boxed{\text{extrema}}$

Mathematically, this usually refers to a maximum or minimum, which can be identified through the use of calculus.

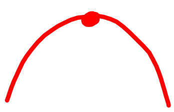
Ex.1 A piece of sheet metal, 60 cm by 30 cm, will be used to make a box with an open top. Determine the dimensions that will maximize volume.





$V = lwh$

$V(x) = (60-2x)(30-2x)(x)$



$V'(x) \longrightarrow$ iRoC of volume wrt x

$\longrightarrow m_{tan}$

$V(x) = (60 - 2x)\underbrace{(30-2x)(x)}_{(30x - 2x^2)}$

① expand

② expand one bracket pair → $V(x) = (\ )(\ )$

③ product rule $V(x) = f[g \cdot h]$

$V'(x) = f'[gh] + f[gh]' \rightarrow g'h + gh'$

$= \frac{df}{dx}(gh) + f \cdot \frac{d(gh)}{dx}$

$= f'gh + f[g'h + gh']$

$= f'gh + fg'h + fgh'$

$V(x) = (60-2x)(30-2x)(x)$

$V'(x) = -2(30-2x)(x) + (60-2x)(-2)(x) + (60-2x)(30-2x)(1)$

$= -60x + 4x^2 - 120x + 4x^2 + 1800 - 120x - 60x + 4x^2$

$= 12x^2 - 360x + 1800$

set $V'(x) = 0$

$\frac{0}{12} = \frac{12x^2 - 360x + 1800}{12}$

$0 = x^2 - 30x + 150$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

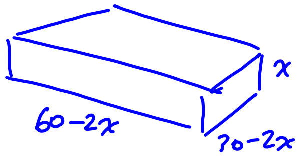
$\vdots$

$x \doteq 23.660$ or $x \doteq 6.340$

$x \doteq 23.660$ or $x \doteq 6.340$

Domain: all dimensions must be positive

$x > 0$ ✓



$60 - 2x > 0$

$60 > 2x$

$2x < 60$

$x < 30$ ✓

$30 - 2x > 0$

$30 > 2x$

$2x < 30$

$x < 15$ reject $x \doteq 23.660$.

$\therefore x \doteq 6.340$

$60 - 2(6.340) \doteq 47.32$

$30 - 2(6.340) \doteq 17.32$

$\therefore$ dimensions to maximize volume are 6.3 cm by 17.3 cm by 47.3 cm

Strategy for solving optimization problems:

1. Read the problem carefully. Determine a function of the independent variable that represents the quantity to be optimized (the dependent variable).

2. Draw a diagram (if possible).

3. Determine the domain of the function.

4. Compare all extreme values and end points to find absolute maximum or minimum values.

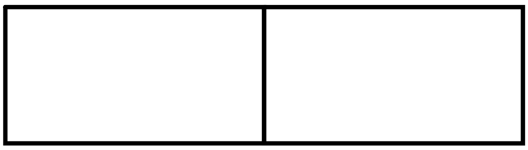
5. Answer the original problem.

Assigned Work:

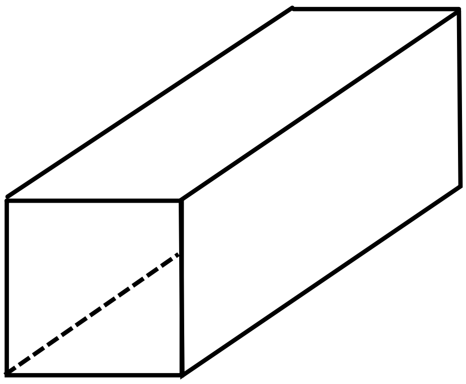
p.145 # 1, 3-12

Ex1: p.145 #4

Ex2: p.145 #7



Ex3: p.145 #8



Ex4: p.146 #10