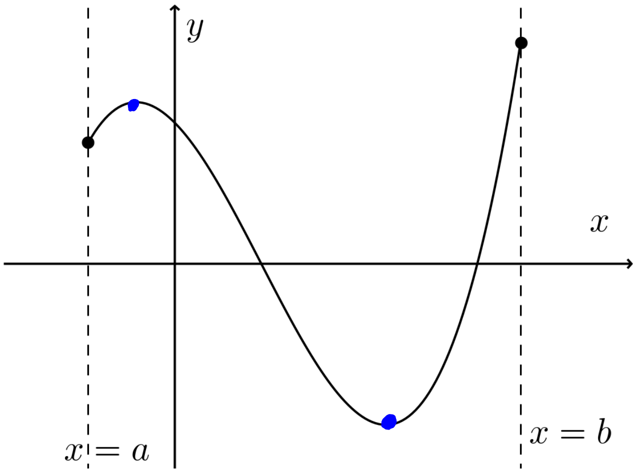
Extreme Values on an Interval

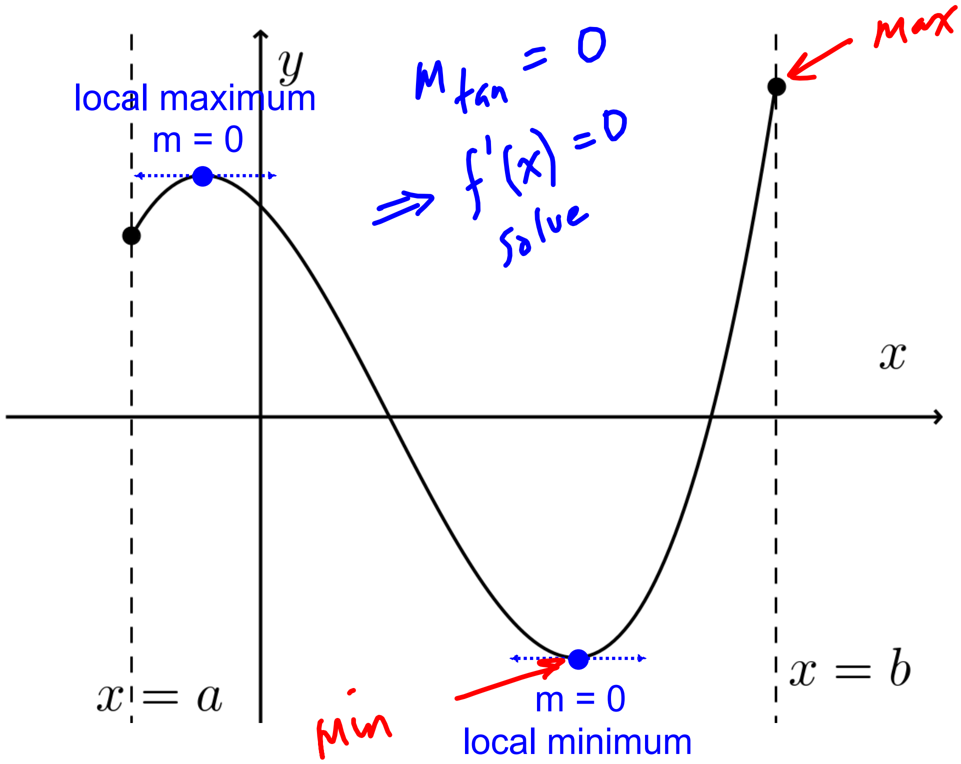
Consider the following section of a cubic polynomial:

(a) Identify the local max and min points on *(a, b)*.

(b) Identify the ~~absolute~~ global max and min points on *[a, b]*.



(a) local extrema - slope of tangent is zero

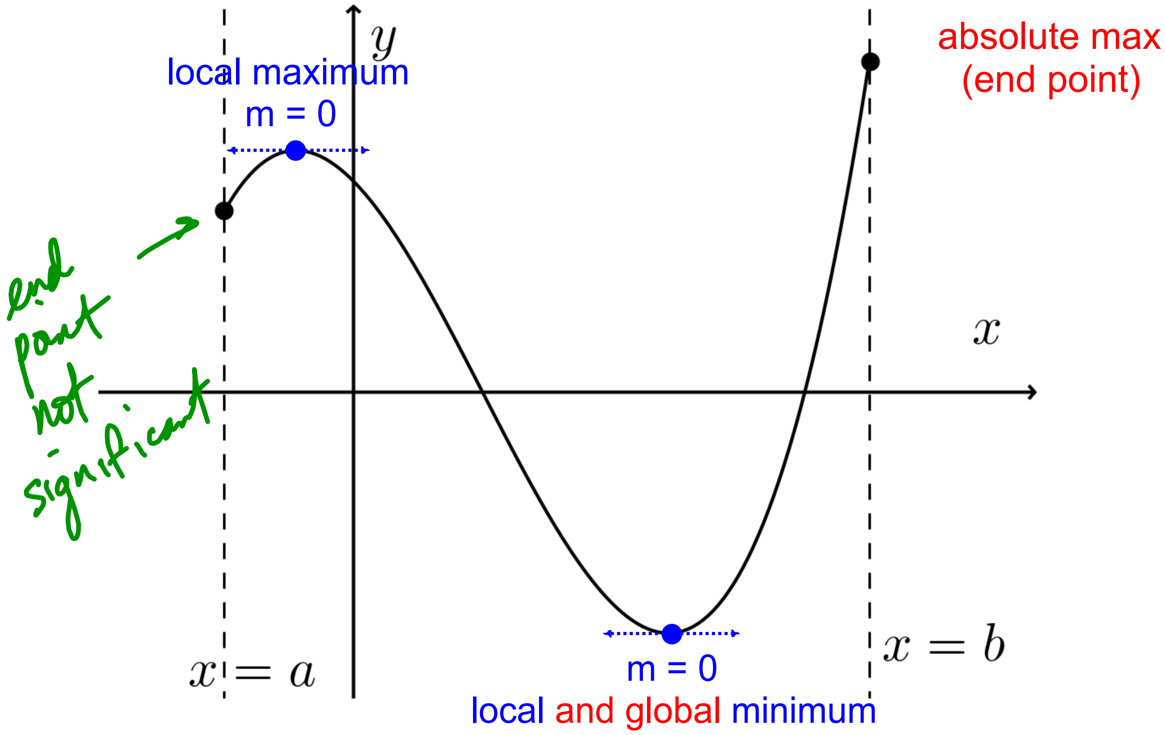


$m_{tan} = 0$

$\Rightarrow f'(x) = 0$

solve

(a) absolute/global extrema - high and low y-values



Extreme values are the minimum and maximum values of a function on a given interval.

Interval notation: $a \leq x \leq b$ or $[a, b]$

To find extreme values:

$M_{tan} = 0$ → solve for x → sub x

(1) Solve $f'(x) = 0$ on the interval, and evaluate $f(x)$ at each solution.

↓ y-values ✓

$f(a) = ?$ ✓ $f(b) = ?$ ✓

(2) Evaluate $f(x)$ at the end points.

(3) Compare the results from steps 1 and 2.

→ absolute extrema

Ex.1 Find the extreme values of

$$f(x) = 3x^4 - 4x^3 - 36x^2 + 20, \; x \in [-3, 4]$$

(a) local :

$f'(x) = 12x^3 - 12x^2 - 72x$

$= 12x(x^2 - x - 6)$

$= 12x(x-3)(x+2)$

Set $f'(x) = 0$

$12x(x-3)(x+2) = 0$

$x = 0, 3, -2$

$f(0) = 20$

$f(3) = -169$

$f(-2) = -44$

y-values

local extrema

$(0,20), (3,-169), (-2,-44)$

min

(b) end points $f(-3) = 47$ $f(4) = -44$

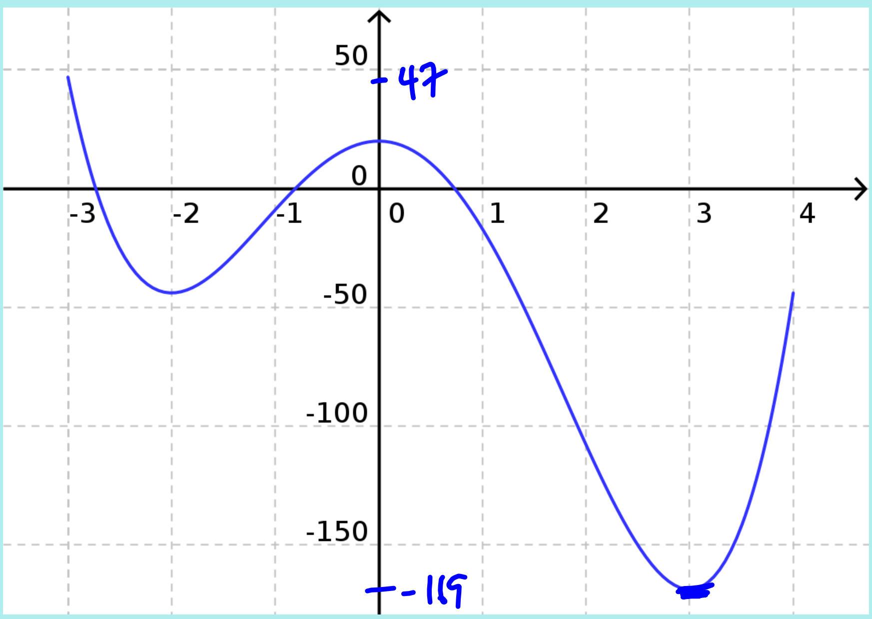
max

$\hookrightarrow (-3, 47)$ $(4, -44)$

$\therefore$ the absolute maximum is 47 and minimum is -169. *

Ex.1 Find the extreme values of

$$f(x) = 3x^4 - 4x^3 - 36x^2 + 20, \; x \in [-3, 4]$$



Ex.2 The amount of light intensity on a point is given by

$$I(t) = \frac{t^2 + 2t + 16}{t+2}, \quad t \in [0, 14]$$

where I is measured in candela, and t in seconds.

(a) Determine the times of extreme intensity.
(b) What is the minimal intensity?

$$I'(t) = \frac{(2t+2)(t+2) - (t^2+2t+16)(1)}{(t+2)^2}$$

$$= \frac{2t^2+4t+2t+4-t^2-2t-16}{(t+2)^2}$$

$$= \frac{t^2+4t-12}{(t+2)^2}$$

$$I'(t) = \frac{(t+6)(t-2)}{(t+2)^2}$$

set $I'(t)=0$, $\frac{(t+6)(t-2)}{(t+2)^2} = 0$ $\quad t \neq -2$

$(t+6)(t-2)=0$

$t = \cancel{-6}, 2$

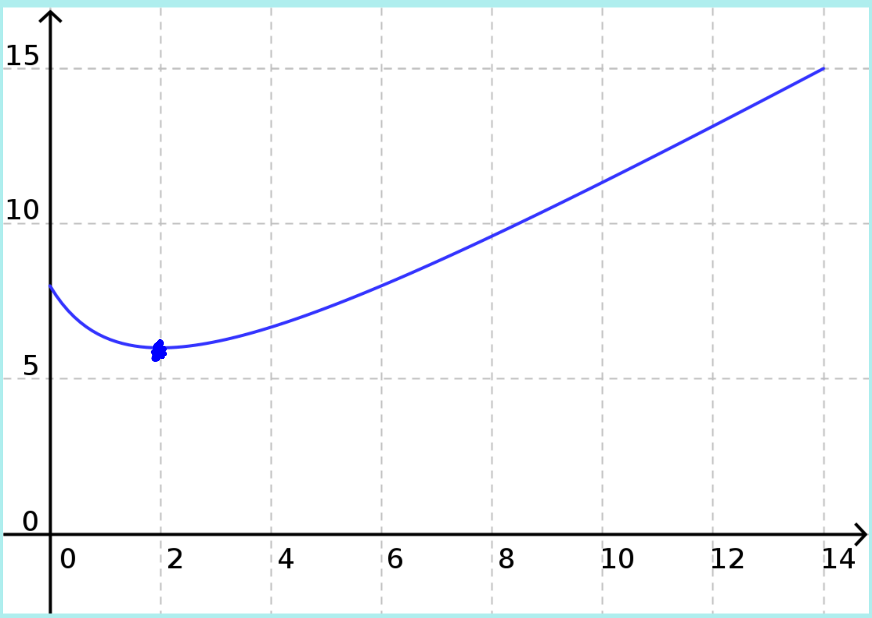
reject

$I(2) = 6$ min $\quad I(0) = 8 \quad I(14) = 15$ max

$\therefore$ times of extreme intensity are 2 seconds (min) and 14 seconds (max).

$$I(t) = \frac{t^2 + 2t + 16}{t+2}, \quad t \in [0, 14]$$

(a) Determine the times of extreme intensity.
(b) What is the minimal intensity?



Assigned Work:

p.117 # 7, 8 (interval notation)
p.136 # 2ab, 3d, 4ab, 6, 7, 8, 9