Unit 2: Applications of Derivatives

March 9/2018

Position, Velocity, and Acceleration

Higher-order derivatives:

Given $y = f(x)$

The first derivative is $\dfrac{dy}{dx} = y' = f'(x)$

The second derivative is $\dfrac{d^2y}{dx^2} = y'' = f''(x)$

$f(x)$

$\frac{d}{dx} f(x) = f'(x)$

$\frac{d}{dx}\left[\frac{d}{dx} f(x)\right] = \frac{d^2}{dx^2} f(x) = f''(x)$

We will start by exploring position, velocity, and acceleration in one-dimension.

1. Position is the distance from the origin, including a direction (positive or negative value).

$$s(t) \text{ or } h(t)$$

2. Velocity is the rate of change of position.

$$v(t) = s'(t) = \frac{ds}{dt}$$

3. Acceleration is the rate of change of velocity.

$$a(t) = v'(t) = s''(t) = \frac{d^2s}{dt^2}$$

$d(t) = v_1 t + \frac{1}{2} a t^2$

$d'(t) = v_1 + at$

$v(t) = v_1 + at$

$v_2 = v_1 + at$

$v'(t) = a$

$a = a$

In this class, we will be working in a single dimension (up/down, left/right, north/south, etc.) at a time.

One direction is made postive (e.g., up, right, north) and the other becomes negative.

Read any questions carefully to see if they have already chosen a 'positive' direction.

For example, if an equation is provided (e.g., height of an object), the direction is already set.

$$h(t) = 2 + 20t - 5t^2$$

Ex.1 The function $s(t) = t(t-3)^2$ describes the position of an object moving along a straight line, $t \geq 0$, in metres at time t, in seconds.

a) Determine the velocity and acceleration at any time t.
b) Find the velocity and acceleration at 4 seconds.
c) Describe the motion of the object after 4 seconds.

(a) $s(t) = t(t-3)^2$

$s'(t) = (1)(t-3)^2 + t\left[2(t-3)^1(1)\right]$

$= (t-3)^2 + 2t(t-3)$

$= (t-3)\left[(t-3) + 2t\right]$

$= (t-3)(3t-3)$

$v(t) = 3(t-3)(t-1)$

$v'(t) = 3\left[(1)(t-1) + (t-3)(1)\right]$

$= 3\left[2t-4\right]$

$a(t) = 6(t-2)$

(b) $v(4) = 3(4-3)(4-1)$
$= 3(1)(3)$
$= 9$
∴ velocity at 4 sec is 9 m/s

$a(4) = 6(4-2)$
$= 12$
∴ acceleration is $12\ m/s^2$.

(c) After 4 seconds, the object is getting farther away (positive velocity) and speed is increasing (positive acceleration)

Ex.2 (see p.125) A baseball is hit vertically upward. The position above the ground (in metres) with respect to time (in seconds) is given by:

$$s(t) = -5t^2 + 30t + 1$$

(a) Determine the maximum height.
(b) Determine the velocity when caught 1 m above ground.

(a) $s'(t) = -10t + 30$

for maximum (extrema), set $s'(t) = 0$

$0 = -10t + 30$
$10t = 30$
$t = 3$

* in some cases, we will need to classify extrema, but here there is only one, and it must be a max.

$s''(t) = -10$
$s''(3) = -10$
concave down → max

$s(3) = -5(3)^2 + 30(3) + 1$
$= -5(9) + 90 + 1$
$= 46$

∴ max height is 46m

(b) set $s(t) =$

Assigned Work:

review "Need to Know" on p.126

p.127 #4, 5, 6ac, 8, 9, 10, 11, 12, 13b, 16